Formal Logic — Exercise Sheet 12

Exercise 45: (Relations and directed graphs)
Visualise the following relations as directed graphs $G = (W, R)$. I.e., the nodes of G are the elements of W, the edges of G are the (ordered!) elements of R. Visualise also (W, R^2) and (W, R^3) in each case.

(a) $W = \{0, 1, 2, 3\}$, $R = \{(n, m) \mid n, m \in W, |n - m| = 1\}$

(b) $W = \{0, 1, 2, 3, 4, 5\}$, $R = \{(n, m) \mid n, m \in W, n + m = 0 \text{ mod } 3\}$

(c) $W = \{\varnothing, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}\}$, $R = \{(n, m) \mid n, m \in W, n \subseteq m\}$

State for each of the nine relations (W, R^i) $(i = 1, 2, 3)$ whether they are reflexive, and/or symmetric, and/or transitive.

Exercise 46: (More rules)
(a) Prove Rule 5 of Theorem 4.1.
(You may use rules 1,2,6 and 7 in order to transform one of the formulas into the other.)

(b) Prove Rule 4 of Theorem 4.1 by showing that $\Box(F \Rightarrow G) \Rightarrow (\diamond F \Rightarrow \diamond G)$ is a tautology.
(You may use rules 1,2,6 and 7 in order to tranform this expression into one like $F \vee \neg F \vee ...$; this is obviously a tautology.)

(c) Show that the following variant of Rule 6 in Theorem 4.1:

$$(\diamond F \wedge \diamond G) \equiv \diamond(F \wedge G)$$

does not hold; for instance, by providing a counterexample.

Exercise 47: (Tautologies)
Two out of the following four formulas are tautologies. Which one is, which one is not a tautology? For the tautologies: prove that they are tautologies. For the ones that are not tautologies give a structure $\mathcal{A} = (W, R, \alpha)$ and $s \in W$ such that $\mathcal{A}(H_i, s) = 0$.

(a) $H_1 = \Box F \Rightarrow \diamond F$

(b) $H_2 = F \Rightarrow \diamond F$

(c) $H_3 = \Box F \Rightarrow \Box\Box F$

(d) $H_4 = \diamond \diamond F \Rightarrow \diamond F$

Hand in your solutions until 20.1.2020 at 11:00 in post box 2183 in V3,
or via email to your tutor.

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