

Formal Logic — Exercise Sheet 13**Exercise 48: (According to rank)**

Determine the modal rank of the following formulas.

- (a) $F_1 = \Box(\Box A \Rightarrow \Box \Diamond B)$
- (b) $F_2 = \Box A \Rightarrow \Box \Diamond(\Box \Diamond A \wedge \neg B)$
- (c) $F_3 = \Box \neg \Diamond(A \vee \neg \Box(A \vee \Diamond \neg B))$
- (d) $F_4 = \Box \Diamond \Box(\neg \Box B \wedge \Diamond(\Diamond \neg \Box B \vee \neg \Diamond A) \wedge \Box \Box \Diamond A)$

Exercise 49: (More tautologies and non-tautologies)

Use the tableau calculus for parts (a) and (b). Recall that a formula is a tautology if and only if its negation is unsatisfiable.

- (a) Prove Rule 4 of Theorem 4.1 by showing that $\Box(F \Rightarrow G) \Rightarrow (\Diamond F \Rightarrow \Diamond G)$ is a tautology.
- (b) Show that $\Diamond \Diamond F \Rightarrow \Diamond F$ is satisfiable, but it is not a tautology. (*Compare Ex. 47 (d)*)
- (c) If we require the frame for $\Diamond \Diamond A \Rightarrow \Diamond A$ to be transitive, would the formula become a tautology?

Exercise 50: (Meaning of closed paths)

Assume you applied the tableau calculus to a formula F in modal logic until it stops. (That is, all vertices $s \Vdash G$ that can possibly be marked are indeed marked now.)

- (a) If the tableau contains no closed path, is F a tautology? Justify your answer by a proof or a counterexample.
- (b) If the tableau is closed (i.e., contains closed paths only), is $\neg F$ satisfiable? Justify your answer.

Exercise 51: (Dr Who shifting levers)

Again Dr Who encounters a device that might possibly explode, thus destroying all life, time and so on. It has three levers marked A , B and C . All three can be shifted either up or down, but the levers interact with each other. For instance, it is always true that not both of lever A and lever B can be up. It is also always true that if lever C is up then lever A is up or lever B is down (or both). At each time holds that if lever C is up then lever B is up, too.

The device explodes as soon as lever C is up at some time. Translate the above conditions into formulas F_1, F_2, F_3 in modal logic and use the tableau calculus in order to show that the device never explodes ...destroying life, time, universe and everything... no matter which levers are shifted by Dr Who (by showing that $F_1 \wedge F_2 \wedge F_3 \wedge \Diamond C$ is unsatisfiable).

Hand in your solutions until 27.1.2020 at 11:00 in post box 2183 in V3,
or via email to your tutor.

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