Dr. D. Frettlöh 12.10.2021

Formal Logic — Exercise Sheet 1

Exercise 1: (Satisfiable, tautology, equivalent)
Consider the formulas

$$F_1 = (A \wedge \neg B) \vee C, \quad F_2 = A \Rightarrow (B \Rightarrow C), \quad F_3 = (A \Rightarrow B) \Rightarrow C$$

Decide for each of them: is the formula a tautology? Is the formula satisfiable? If it is, give a satisfying valuation $\mathcal{A}$ for the respective formula. Are F_1 and F_2 equivalent? Are F_1 and F_3 equivalent? Are F_2 and F_3 equivalent?

Exercise 2: (Disney™ Princesses™)
(a) The three Disney™ princesses™ Arielle™, Belle™ and Cinderella™ are invited to a party. They make the following statements:

Arielle™ :	If Cinderella™ will not come to the party then I will come to the party.
Belle™:	If Arielle™ will come to the party then I will not come.
Cinderella™:	I will come to the party if Belle™ and Arielle™ will come as well; otherwise I will not come.

Translate the statements into one single formula F in propositional logic. Is F satisfiable? If so, give a valuation $\mathcal{A}$ such that $\mathcal{A} \models F$.

(b) Do the same for the following statements.

Arielle™:	I will not come to the party.
Belle™:	I will come to the party if Cinderella™ will not come, and I will not come to the party if Cinderella™ will come.
Cinderella™:	I will come to the party if Belle™ or Arielle™ (or both) will come as well; otherwise I will not come.

Exercise 3: (Truth tables upside down))
Let F be a formula. Let F' be the formula arising from F by replacing all atomic formulas by their negations. That is, each A in F is replaced by $\neg A$, each B in F is replaced by $\neg B$ and so on (hence $\neg A$ in F becomes $\neg\neg A$ in F'). Show that the truth table of F' is obtained from the truth table of F by turning the column under F upside down.

(For this the truth table needs to be in the correct binary order; for instance 000, 001, 010, 011, 100, 101, 110, 111 in the case of three atomic formulas.)

Exercise 4: (Sufficient sets of operators)
(a) Prove that for each formula F there is an equivalent formula G using only $\neg$ and $\Rightarrow$.
(Hint: if you can express $\wedge$ and $\vee$ by combinations of $\neg$ and $\Rightarrow$ then you are done.)

(b) Prove that there is some formula F such that there is no formula G equivalent to F, where G uses only $\wedge$ and $\oplus$. *($\oplus$ means XOR, the exclusive or, see lecture notes).*

(c) Prove that for each formula F there is an equivalent formula G using only $\Rightarrow$ and $\oplus$.

Send your solutions until Tue 19.10.2021 at 14:00 to your respective tutor.

Please indicate the name of the tutor on your solution sheet.

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