Dr. D. Frettlöh 19.10.2021

Formal Logic — Exercise Sheet 2

Exercise 5: (Laws of Logic)
(a) Prove the following laws using truth tables, compare Theorem 1.1.

1. $\neg(F \vee G) \equiv \neg F \wedge \neg G$ (de Morgan's law 2)
2. $F \wedge (G \vee H) \equiv (F \wedge G) \vee (F \wedge H)$ (Distributivity 1)

(b) Show the following equivalences using the laws in Theorem 1.1 and the definitions of $\Leftrightarrow$ and $\Rightarrow$ only (compare Example 1.4). For each transformation state the law you used.

1. $F \Rightarrow G \equiv \neg G \Rightarrow \neg F$ (contraposition)
2. $F \wedge (G \Leftrightarrow H) \equiv (F \wedge G \wedge H) \vee \neg(\neg F \vee G \vee H)$

Exercise 6: (CNF and DNF)
Transform the following formulas into conjunctive normal form and into disjunctive normal form, using Algorithm 1.1 shown in the lecture.

$$F = A \wedge \neg\big(B \vee (C \wedge D)\big), \qquad G = \neg\big(A \Rightarrow (B \Leftrightarrow C)\big)$$

Exercise 7: (satisfiable vs tautology)
Prove or give a counterexample:

(a) If F is satisfiable and $F \Rightarrow G$ is satisfiable, then G is satisfiable.

(b) If F is a tautology and $F \Rightarrow G$ is a tautology, then G is a tautology.

(c) If F is satisfiable and $F \Rightarrow G$ is a tautology, then G is satisfiable.

(d) If F is satisfiable and $F \Rightarrow G$ is a tautology, then G is a tautology.

Exercise 8: (Switch and and or)
Let $F \equiv G$. Let neither F nor G contain any $\Leftrightarrow$ or $\Rightarrow$. Let F' (respectively G') be the resulting formulas if one changes each $\vee$ in F (respectively G) into $\wedge$ and vice versa. Prove that $F' \equiv G'$.

Send your solutions until Tue 26.10.2021 at 14:00 to your respective tutor.

Please indicate the name of the tutor on your solution sheet.

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