Dr. Dirk Frettlöh 26.10.2021

Formal Logic — Exercise Sheet 3

Exercise 9: (Horn formula algorithm)
(a) Apply the Marking Algorithm for Horn formulas to the following two formulas F and G. Is F (resp. G) satisfiable? If yes, please give all valuations $\mathcal{A}$ with $\mathcal{A} \models F$ (resp. $\mathcal{A} \models G$).

$$F = (A \wedge B \Rightarrow C) \wedge (B \wedge D \wedge E \Rightarrow C) \wedge (1 \Rightarrow A) \wedge (A \Rightarrow B) \wedge (D \wedge C \Rightarrow E) \wedge (1 \Rightarrow E) \wedge (D \Rightarrow 0)$$

$$G = (\neg A_1 \vee \neg A_2 \vee \neg A_3 \vee A_4) \wedge (\neg A_1 \vee \neg A_3 \vee A_6) \wedge \neg A_6 \wedge A_4 \wedge (\neg A_4 \vee \neg A_5 \vee A_1) \wedge (\neg A_1 \vee \neg A_2 \vee A_3) \wedge (\neg A_5 \vee \neg A_1 \vee A_2) \wedge (A_5 \vee \neg A_4)$$

(b) The five™ Disney™ princesses™ Arielle™, Belle™, Cinderella™, Diana™ and Eliza™ are invited to a party™. Again they state strict opinions:

- Arielle™: If Eliza™ and Diana™ are coming to the party I will come, too.
- Belle™: If Cinderella™ is coming I will come as well.
- Cinderella™: I will go to the party anyway.
- Diana™: If Eliza™ and Cinderella™ will come I will come, too.
- Eliza™: If Belle™ and Cinderella™ are coming I will come, too.

Translate their statements into a single Horn™ formula F. Is F satisfiable? If yes, please give a valuation $\mathcal{A}$ with $\mathcal{A} \models F$. What is the minimal satisfying valuation?

Exercise 10: (Borromean formulas)
(a) Find three formulas F_1, F_2, F_3 such that $F_i \wedge F_j$ is satisfiable for all choices of $1 \leq i < j \leq 3$, but $F_1 \wedge F_2 \wedge F_3$ is not satisfiable.

(b) Find four formulas F_1, F_2, F_3, F_4 such that $F_i \wedge F_j \wedge F_k$ is satisfiable for all choices of $i, j, k \in \{1, 2, 3, 4\}$, but $F_1 \wedge F_2 \wedge F_3 \wedge F_4$ is not satisfiable.

Exercise 11: (Easy decisions)
(a) Show that any Horn formula F (in CNF) is satisfiable if each disjunctive clause contains at least one $\neg$.

(b) State a formula that does not have an equivalent Horn formula. (Congratulations if you found one: you just proved that not each formula has an equivalent Horn formula.)

Exercise 12: (Infinitely many formulas)
Find all valuations for $A_1, A_2, \ldots$ satisfying the infinite set of formulas

$$M = \{A_1 \vee \neg A_2, A_2 \vee \neg A_3, A_3 \vee \neg A_4, A_4 \vee \neg A_5, A_5 \vee \neg A_6, \ldots\}$$

(Hint: there are more than three.) Give a reason why your answer includes all satisfying valuations.

Send your solutions until Tue 2.11.2021 at 14:00 to your respective tutor.

Please indicate the name of the tutor on your solution sheet.

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