Dr. D. Frettlöh 9.11.2021

Formal Logic — Exercise Sheet 5

Exercise 17: (Testing tautology and consequence via resolution)
(a) Using resolution, show that

$$F = (\neg B \wedge \neg C \wedge D) \vee (\neg B \wedge \neg D) \vee (C \wedge D) \vee B$$

is a tautology.

(b) Using resolution, show that $A \Rightarrow D$ is a consequence of the set of formulas

$$F = \{A \Rightarrow B, B \Rightarrow C, C \Rightarrow D\}.$$

Exercise 18: (The Dr Who diet)
Dr Who is asked how he reached his remarkable age. His answer: "A strict diet. Whenever I eat something the following rules apply: When I drink alcohol to the meal I also eat fish fingers. Whenever I don't eat beans I eat carrots. When I eat fish fingers and carrots I will also have beans. Whenever I eat carrots I also drink alcohol."

Model the situation into a formula in propositional logic. Use the resolution method to show that Dr Who eats beans whith each meal.

Exercise 19: (Resolution variants)
For each of the following statements, find out whether they are true or false. If "true" give a convincing reason why. If "false" provide a counterexample.

(a) Clauses of the form $\{A, \neg A, B, \ldots\}$ can be reduced to $\{B, \ldots\}$.

(b) The resolution calculus is still correct if we allow for the simultaneous resolution of two literals. That means for instance $\{A, B, C\}, \{\neg A, \neg B, C\}$ yields $\{C\}$.

(c) The resolution calculus is still correct if we allow for the simultaneous resolution of two literals wit the same atomic formula. That means for instance $\{A, \neg A, C\}, \{A, \neg A, C\}$ yields $\{C\}$.

(d) Clauses of the form $\{A, \neg A, B, \ldots\}$ can just be ignored: omitting them yields the same results.

Exercise 20: (Efficient resolution calculus)
In the worst case showing the satisfiability of a CNF needs exponentially many steps (for instance, resolutions). But there are many instances where it is efficient. Here are two.

(a) Show that a formula in CNF is always satisfiable if each clause contains at least one negative literal. Is the same true if we replace "negative" with "positive"?

(b) Let F be a formula in CNF using n different atomic formulas. Show that in the resolution algorithm the number of steps is at most $O(n^2)$ if all clauses in the CNF of F have length at most two. Here "one step" is taking one resolvent.
(This shows essentially that the runtime for 2SAT is $O(n^2)$, i.e., 2SAT is efficiently decidable.)

Send your solutions until Tue 16.11.2021 at 14:00 to the tutor who sent you the correction of your last solutions.

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