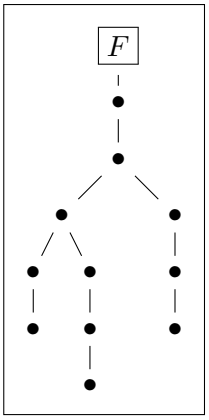
Dr. D. Frettlöh 16.11.2021

Formal Logic — Exercise Sheet 6

Exercise 21: (Dr Who pressing buttons)
Dr Who encounters some device about to explode, thus destroying all life, time and space existing. It has three buttons: A, B and D. The device can be shut down (hence not explode) by pressing the correct buttons. It is known that if button B is pressed then button A must be pressed, too. Pressing both buttons B and D leads to immediate disaster. If button B is not pressed then button D must be pressed. Moreover, if button D is not pressed then A needs to be pressed.

Translate the situation above into a single formula in propositional logic and apply the tableau calculus in order to (a) decide whether Dr Who is able to save the universe (i.e., whether the formula is satisfiable), and (b) find a correct combination of buttons (i.e., a satisfying valuation). How many correct combinations are there?

Exercise 22: (Find the formula)
Find a formula F that, when the tableau calculus is applied to F, yields the tree in the image on the right . Are there different formulas yielding this tree? For the latter question, we consider formulas as „not different“ if they arise by just renaming the atomic formulas. This is, $A \vee B$ is not really equal to $B \vee A$, or to $B \vee C$. But for this question we consider them as not different, since they are equal up to renaming the letters.



Exercise 23: (Meaning of closed paths)
(a) If the tableau calculus applied to F yields a tableau with no closed path, is F a tautology? Justify your answer.
(b) If the tableau calculus applied to $\neg F$ yields a tableau with closed paths only, is F a tautology? Justify your answer.

Exercise 24: (Equivalence relations)
Let B denote the set of all people living in Bielefeld. Which of the following relations are equivalence relations (on the corresponding sets)? Give a convincing reasion why they are, or provide a counterexample. For the ones that are equivalence relations: list all equivalence classes.

1. $R = \varnothing$ on B.
2. $R = \{(a, a) \mid a \in B\}$ on B.
3. $R = \{(a, b) \mid a, b \in \mathbb{N},\ a \geq b\})$ on $\mathbb{N}$.
4. $R = \{(a, b) \mid a, b \in \mathbb{N},\ |a - b| \leq 3\})$ on $\mathbb{N}$.
5. $R = \{(a, b) \mid a, b \in \mathbb{N},\ a \bmod 10 = b \bmod 10\})$ on $\mathbb{N}$.
6. $R = \{(a, b) \mid a, b \in B,\ a$ and b have the same birthday$\}$.

Send your solutions until Tue 23.11.2021 at 14:00 to the tutor who sent you the correction of your last solutions.

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