Formal Logic — Exercise Sheet 7

Exercise 25: (Models?)
Which of the following structures are models for

$$F = \exists x\, \exists y\, \exists z\, \big(P(x,y) \wedge P(z,y) \wedge P(x,z) \wedge \neg P(z,x)\big),$$

which are not? Please give a convincing reason for each of your answers.

(a) $U_{\mathcal{A}} = \mathbb{N}$, $P^{\mathcal{A}} = \{(m,n) \mid m,n \in \mathbb{N},\ m < n\}$

(b) $U_{\mathcal{A}} = \mathbb{N}$, $P^{\mathcal{A}} = \{(n,n+1) \mid n \in \mathbb{N}\}$

(c) $U_{\mathcal{A}} = \{f : \mathbb{R} \to \mathbb{R}\}$ (all functions from $\mathbb{R}$ to $\mathbb{R}$), $P^{\mathcal{A}} = \{(f,g) \mid f = g'\}$

(d) $U_{\mathcal{A}} =$Pot($\mathbb{N}$), (that is the set of all subsets of $\mathbb{N}$), $P^{\mathcal{A}} = \{(A,B) \mid A,B \subset \mathbb{N},\ A \subseteq B\}$

Exercise 26: (Structures and models)
List all partial formulas of the following formulas F and G, and all terms, and write down the matrix of both F and G.

$$F = \exists x\, P(x,f) \wedge \forall y\, Q(y)$$

$$G = P(x,a) \vee \forall x\, \exists y\, \big(Q(x,f(y)) \wedge P(x,y)\big)$$

Find a structure that is a model for F, and another structure that is not a model for F. Do the same for G.

Exercise 27: (Small universes)
(a) Find a formula F of first-order logic without free variables such that F is satisfiable only if $U_{\mathcal{A}}$ has at least three elements. (I.e., F is unsatisfiable for all $U_{\mathcal{A}}$ where $U_{\mathcal{A}}$ has only one or two elements.)

(b) Find a formula F of first-order logic with identity (see Remarks 2.1 and 2.4), and without free variables, such that for all $\mathcal{A}$ with $\mathcal{A} \models F$ holds that $U_{\mathcal{A}}$ has at most two elements.

Exercise 28: (Easy decisions)
Prove that every predicate logic formula that only contains $\wedge, \vee, \forall, \exists, \Rightarrow$, variables and predicate symbols is satisfiable. Is such a formula also valid? What if we allow in addition function symbols?

Send your solutions until Tue 30.11.2021 at 14:00 to the tutor who sent you the correction of your last solutions.

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