Dr. D. Frettlöh 30.11.2021

Formal Logic — Exercise Sheet 8

Exercise 29: (Skolemification and Herbrandization)
(a) Establish the Skolem normal form G of

$$F = \neg\exists x\,\forall y\,\big(P(y) \vee Q(g(a,x))\big) \wedge \neg\forall x\big(P(b) \wedge \neg Q(f(x))\big),$$

(b) Write down 12 terms of the Herbrand universe $H(G)$ of the Skolem normal form G of F.

(c) Write down 12 terms of the Herbrand universe $H(F)$ of F.

Exercise 30: (Not a law)
Show that the two formulas in (a) (respectively, in (b)) are not equivalent to each other by providing a (counter-)example for each.

(a) $\exists x\,\forall y\,P(x,y) \quad \not\equiv \quad \forall y\,\exists x\,P(x,y)$

(b) $(\exists x\,F) \wedge (\exists x\,G) \quad \not\equiv \quad \exists x(F \wedge G)$.

Exercise 31: (Equivalence)
Let

$$F = \neg\forall x\,\exists y\,\big(P(x) \vee \neg Q(y)\big) \vee \exists z\,\exists x\,R(x,z)$$

and

$$G = \exists z\,\exists x\,\forall y\,\Big(\big(\neg P(x) \vee R(x,z)\big) \wedge \big(Q(y) \vee R(x,z)\big)\Big).$$

Show that $F \equiv G$ by applying the transformation rules of Theorem 2.1.

Exercise 32: (Equisatisfiablity)
Show why step 4 in Algorithm 2.1 does not preserve equivalence. (Compare Theorem 2.4 and the remark after Algorithm 2.1.)
Hint: a valuation $\mathcal{A}$ for two formulas needs to explain each symbol in both formulas. What new symbols do arise here?

Send your solutions until Tue 7.12.2021 at 14:00 to the tutor who sent you the correction of your last solutions.

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