Dr. D. Frettlöh 7.12.2021

Formal Logic — Exercise Sheet 9

Exercise 33: (All in one)
Establish THE normal form (TNF) of

$$F = \neg \forall x \, \exists y \, \big(P(y,a) \wedge \neg Q(x)\big) \wedge \neg \exists x \big(P(x,x) \Rightarrow Q(x)\big),$$

determine the Herbrand universe of the TNF, and apply the resolution calculus to it. Is F satisfiable?

Exercise 34: (Dancing paradox)
Consider this statement:

At this (nonempty) party there is someone such that, if this person is dancing, then everyone at the party is dancing.

Translate the statement above into a formula F in first-order logic. Take $U_{\mathcal{A}}$ as “all people at this party”. Prove that the statement is true for any (nonempty) party by showing that it is valid. In order to do this show that $\neg F$ is unsatisfiable: establish the THE normal form (TNF) of $\neg F$ and apply the resolution calculus to it.
(Since the statement is true it is not really a paradox; it only sounds like one.)

Exercise 35: (I can't get no satisfaction)
Establish THE normal form (TNF) of

$$F = \forall x \, \Big(\big(\exists y (P(a) \Rightarrow Q(y))\big) \wedge (Q(x) \Rightarrow P(x)) \Big)$$

and apply the resolution calculus. Is F satisfiable?

Exercise 36: (Infinite models)
Prove that there are formulas having infinite models only. That is:

(a) Show that $F = \forall x \, P(x, f(x)) \wedge \forall y \, \neg P(y,y) \wedge \forall x \, \forall y \, \forall z \, \big((P(x,y) \wedge P(y,z) \Rightarrow P(x,z)\big)$ is satisfiable by finding a model for F.

(b) Show that F is not satisfiable for any $\mathcal{A} = (U_{\mathcal{A}}, I_{\mathcal{A}})$ where $U_{\mathcal{A}}$ is finite.

Send your solutions until Tue 14.12.2021 at 14:00 to the tutor who sent you the correction of your last solutions.

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