Dr. D. Frettlöh 14.12.2021

Formal Logic — Exercise Sheet 10

Exercise 37: ("So this dude's his own baby, huh?")
Consider the cartoon on page 17 in the lecture notes. Derive the same conclusion as the little boy by applying resolution calculus with identity. Choose as universe "all humans". Translate the first two statements into formulas F and G and use two constants: a for "me" and b for "baby". Show that $a = b$ is a consequence of the two former staments by showing that $F \wedge G \wedge \neg(a = b)$ is unsatisfiable.

Exercise 38: (Prove the cardinality)
Determine the cardinality of each of the following sets and prove your answer. You may describe the bijections you use in any way, using `if ... then`, or enumerated lists, or functions, or ... You may as well use the Schröder-Bernstein Theorem.

- **(a)** $\mathcal{P}(\mathbb{R})$
- **(b)** $\mathbb{Q} \times \mathbb{Q}$
- **(c)** The set of all sequences $(a_n)_{n\in\mathbb{N}}$ with values $a_n \in \mathbb{R}$.
- **(d)** The set of all functions $f : \mathbb{R} \to \mathbb{R}$.

Exercise 39: (Decidable first-order logic)
(a) Let F be a formula without function symbols. Show that the Herbrand universe $H(F)$ and the Herbrand expansion $E(F)$ are finite. *(Constants are allowed. Function means here and in (b): function symbol with one or more inputs.)*

(b) *Monadic first-order logic* is first-order logic without function symbols, and where all predicates have only one input. Let F be a formula in monadic first-order logic containing n predicates $P_1, \ldots, P_n$. Show that if F is satisfiable, then there is a model $\mathcal{A} = (U_\mathcal{A}, I_\mathcal{A})$ for F such that $U_\mathcal{A}$ has 2^n elements.

(c) Use (b) to show that the resolution calculus applied to a given formula F in monadic first-order always terminates. (That is, the question "is a formula in monadic first order logic satisfiable" is decidable.)

Exercise 40: (Tricky bijections)
Your aim is to show that $|[0, 1]| = |\mathbb{R}|$.

(a) Find a bijection from the closed interval $[0, 1]$ into the open interval $]0, 1[$.

(b) Find a bijection from the open interval $]0, 1[$ into the open interval $]-1, 1[$.

(c) Find a bijection from the open interval $]-1, 1[$ into $\mathbb{R}$.

As in Exercise 38 you may describe the bijections in any appropriate way.

Send your solutions until Tue 21.12.2021 at 14:00 to the tutor who sent you the correction of your last solutions.

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