Formal Logic — Exercise Sheet 11

Exercise 41: (Undecidable problems I: Post correspondence problem)
For each of the following instances of the Post correspondence problem, find a solution for it or show that it has no solution.

(a) $u_1 = 01, u_2 = 10, u_3 = 101$, and $v_1 = 010, v_2 = 1, v_3 = 10$.

(b) $u_1 = 101, u_2 = 1, u_3 = 00$ and $v_1 = 1, v_2 = 01, v_3 = 10$.

(c) $u_1 = 1, u_2 = 10, u_3 = 01, u_4 = 10$ and $v_1 = 11, v_2 = 0, v_3 = 010, v_4 = 1$.

(d) $u_1 = 101, u_2 = 11, u_3 = 011$ and $v_1 = 1, v_2 = 0110, v_3 = 101$.

Exercise 42: (Undecidable problem II: Mortal Matrices)
Decide which of the following sets of matrices are sets of mortal matrices. Give either an example for a product yielding the zero matrix, or give a convincing reason why this is not possible.

(a) $A_1 = \left(\begin{smallmatrix} 1 & 1 \\ 0 & 1 \end{smallmatrix}\right)$, $A_2 = \left(\begin{smallmatrix} 1 & 0 \\ 1 & 1 \end{smallmatrix}\right)$, $A_3 = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right)$.

(b) $B_1 = \left(\begin{smallmatrix} 1 & 2 \\ 1 & 0 \end{smallmatrix}\right)$, $B_2 = \left(\begin{smallmatrix} -2 & -2 \\ 1 & 1 \end{smallmatrix}\right)$, $B_3 = \left(\begin{smallmatrix} 1 & 0 \\ 2 & 0 \end{smallmatrix}\right)$.

(c) $C_1 = \left(\begin{smallmatrix} 1 & 1 \\ 2 & 2 \end{smallmatrix}\right)$, $C_2 = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right)$, $C_3 = \left(\begin{smallmatrix} 1 & 0 \\ 1 & 2 \end{smallmatrix}\right)$.

Exercise 43: (Undecidable problem III: Wang tiles)
For each of the two collections of four Wang tiles below, prove that they can tile the plane (according to the rules: squares are placed vertex-to-vertex, adjacent edges carry the same colour, tiles are not rotated or reflected), or show that there is no such tiling.

Exercise 44: (Computational convergence)
Explain point 4 in Subsection 3.2: why is it not true that the limit of each convergent sequence of computable numbers is computable?

Send your solutions until Tue 11.01.2022 at 14:00 to the tutor who sent you the correction of your last solutions.

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