

Formal Logic — Exercise Sheet 12**Exercise 45: (Football experts)**

Formulate the following statements as formulas in modal logic. Use atomic formulas A for Arminia Bielefeld, B for Bayern München, and the same frame as in Example 4.3.

Moreover, show that the four formulas are not simultaneously satisfiable. That is, show that under *each* structure at least one of these statements is wrong.

- (a) “Bayern München will always win the championship from now on”
- (b) “From some year on, Arminia Bielefeld will always win the championship”
- (c) “It will never be the case that both Arminia Bielefeld and Bayern München will win the championship” (in the same year)
- (d) “There will be a year such that from that year on, if Arminia will not win the championship, then Bayern München will not win the championship”

Exercise 46: (Frames)

Given the frame $W = \mathbb{N}$, let R be the relation $<$ (hence $R = \{(n, m) \mid n, m \in \mathbb{N}, n < m\}$), and the valuation $\alpha : \{A, B\} \times \mathbb{N} \rightarrow \{0, 1\}$ be given by

$$\alpha(A, n) = \begin{cases} 1 & \text{if } n \text{ odd} \\ 0 & \text{else} \end{cases} \quad \alpha(B, n) = \begin{cases} 1 & \text{if } n < 5 \\ 0 & \text{else} \end{cases}$$

Determine the truth values of the following formulas in the point $s = 3$.

- (a) $F = \diamond \diamond A$
- (b) $G = \diamond \Box \neg B$
- (c) $F = \diamond \diamond B$
- (d) $H = \diamond(A \wedge \Box \neg B)$

Exercise 47: (Relations and directed graphs)

Visualise the following relations as directed graphs $G = (W, R)$. I.e., the nodes of G are the elements of W , the edges of G are the (ordered!) elements of R . Visualise also (W, R^2) and (W, R^3) in each case.

- (a) $W = \{\text{rock, scissors, paper}\}$, $R = \{(\text{rock,scissors}), (\text{scissors, paper}), (\text{paper, rock})\}$
- (b) $W = \{0, 1, 2, 3\}$, $R = \{(n, m) \mid n, m \in W, |n - m| \bmod 4 = 1\}$
- (c) $W = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}\}$, $R = \{(n, m) \mid n, m \in W, n \subseteq m\}$

State for each of the nine relations (W, R^i) ($i = 1, 2, 3$) whether they are reflexive, and/or symmetric, and/or transitive.

Bonus question without bonus points: how can you see in the graph each of the three properties reflexive, symmetric, transitive?

Exercise 48: (Tautologies?)

Which of the following four formulas are tautologies, which ones are not? For the tautologies: prove that they are tautologies (e.g. by transforming them in to a formula that is obviously a tautology). For the formulas that are not tautologies give a structure $\mathcal{A} = (W, R, \alpha)$ and $s \in W$ such that $\mathcal{A}(H_i, s) = 0$.

- (a) $H_1 = \Box F \Rightarrow \diamond F$
- (b) $H_2 = \diamond F \Rightarrow F$
- (c) $H_3 = \Box F \Rightarrow \Box \Box F$
- (d) $H_4 = \diamond F \Rightarrow \diamond \diamond F$

Send your solutions until Tue 18.1.2022 at 14:00 to the tutor who sent you the correction of your last solutions.

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