

**Formal Logic — Exercise Sheet 13****Exercise 49: (According to rank)**

Determine the modal rank of the following formulas.

- (a)  $F_1 = \diamond(\diamond A \wedge \diamond \Box B)$
- (b)  $F_2 = \diamond A \vee \diamond \neg \Box(\diamond \Box A \wedge \neg B)$
- (c)  $F_3 = \diamond \neg \Box(A \Rightarrow \Box \diamond(A \vee \neg \Box B))$
- (d)  $F_4 = \diamond \Box \neg \diamond(\neg \diamond B \wedge \Box \diamond(\Box \neg \diamond B \vee \neg \Box A) \wedge \diamond \diamond \Box A)$

**Exercise 50: (More tautologies)**

Use the tableau calculus for part (a). Give a convincing reason for your answers in (b) and (c).

- (a) Show that  $F \Rightarrow \diamond F$  is satisfiable, but it is not a tautology.
- (b) If we require the frame for  $F \Rightarrow \diamond F$  to be reflexive, would the formula become a tautology?
- (c) If we require the frame for  $F \Rightarrow \diamond F$  to be transitive, would the formula become a tautology?

**Exercise 51: (More rules)**

Use the tableau calculus for this exercise. Recall that a formula is a tautology if and only if its negation is unsatisfiable.

- (a) Prove Rule 4 of Theorem 4.1 by showing that  $\Box(F \Rightarrow G) \Rightarrow (\diamond F \Rightarrow \diamond G)$  is a tautology.
- (b) Prove Rule 6 of Theorem 4.1 by showing that  $\Box(F \wedge G) \Leftrightarrow \Box F \wedge \Box G$  is a tautology. First eliminate the  $\Leftrightarrow$  by transforming the formula into some equivalent formula that uses only  $\neg, \wedge, \vee, \Box, \diamond$ .

**Exercise 52: (Dr Who shifting levers)**

Again Dr Who encounters a device that might possibly explode, thus destroying all life, time, universes and so on. It has three levers marked  $A$ ,  $B$  and  $C$ . All three can be shifted either up or down, but the levers interact with each other. For instance, it is always true that not both of lever  $A$  and lever  $B$  can be up. It is also always true that if lever  $C$  is up then lever  $B$  is up. Moreover, it is always true that if lever  $A$  is down then lever  $C$  is down, too.

The device explodes as soon as lever  $C$  is up at some time. Translate the above conditions into formulas  $F_1, F_2, F_3$  in modal logic and use the tableau calculus in order to show that the device never explodes ...destroying life, time, universes and everything... no matter which levers are shifted by Dr Who (by showing that  $F_1 \wedge F_2 \wedge F_3 \wedge \diamond C$  is unsatisfiable).

Send your solutions until Tue 25.1.2022 at 14:00 to the tutor who sent you the correction of your last solutions.

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