Dr. D. Frettlöh 11.10.2022

Formal Logic — Exercise Sheet 1

Exercise 1: (Satisfiable, tautology, equivalent)
Consider the formulas

$$F_1 = (A \wedge B) \vee \neg A, \quad F_2 = A \Rightarrow (A \Rightarrow B), \quad F_3 = (A \Rightarrow B) \wedge A \Rightarrow B.$$

Decide for each of them: is the formula a tautology? Is the formula satisfiable? If it is, give a satisfying valuation $\mathcal{A}$ for the respective formula. Are F_1 and F_2 equivalent? Are F_1 and F_3 equivalent? Are F_2 and F_3 equivalent? Is F_3 equivalent to $A \vee \neg A$?

Exercise 2: (Play with XOR))
Find a formula including $\oplus$ that is a tautology; and find a formula including $\oplus$ that is unsatisfiable.

Exercise 3: (Truth tables upside down))
Let F be a formula. Let F' be the formula arising from F by replacing all atomic formulas by their negations. That is, each A in F is replaced by $\neg A$, each B in F is replaced by $\neg B$ and so on (hence $\neg A$ in F becomes $\neg\neg A$ in F'). Show that the truth table of F' is obtained from the truth table of F by turning the column under F upside down.

(For this the truth table needs to be in the correct binary order; for instance 000, 001, 010, 011, 100, 101, 110, 111 in the case of three atomic formulas.)

Exercise 4: (Sufficient sets of operators)
(a) Prove that for each formula F there is an equivalent formula G using only $\neg$ and $\wedge$.

(b) Prove that there is some formula F such that there is no formula G equivalent to F, where G uses only $\wedge$ and $\vee$.

(c) NOR, aka "not or", aka $\downarrow$, is defined by $A \downarrow B = \neg(A \vee B)$. Prove that for each formula F there is an equivalent formula G using only NOR.

(Hint to (a) and (c): if you can express $\neg$ and $\wedge$ and $\vee$ by combinations of the respective operator(s) then you are done.)

Send your solutions until Tue 18.10.2022 at 14:00 to your respective tutor.

Please indicate the name of the tutor on your solution sheet.

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