Dr. D. Frettlöh 18.10.2022

Formal Logic — Exercise Sheet 2

Exercise 5:
Using propositional logic makes it easy to show identities for sets expressed by union, intersection and set difference. For instance, $x \in A \cap B$ means: $x \in A \wedge x \in B$, $x \notin A$ means $\neg(x \in A)$, $x \in A \setminus B$ means $x \in A \wedge \neg(x \in B)$ and so on. Show the following identities by translating them into formulas and showing their equivalence.

$$C \setminus (A \cap B) = (C \setminus A) \cup (C \setminus B), \quad (A \setminus B) \cap C = A \cap (C \setminus B), \quad A \setminus (B \setminus C) = (A \cap C) \cup (A \setminus B)$$

Look up what a *Venn diagram* is and illustrate the sets by their Venn diagrams.

Exercise 6: (CNF and DNF)
Transform the following formulas into conjunctive normal form and into disjunctive normal form, using Algorithm 1.1 shown in the lecture.

$$F = A \vee \neg\big(B \wedge (C \vee D)\big), \qquad G = \neg\big(A \Leftrightarrow (B \Rightarrow C)\big)$$

Exercise 7: (Borromean formulas)
(a) Find three formulas F_1, F_2, F_3 such that $F_i \wedge F_j$ is satisfiable for all choices of $1 \leq i < j \leq 3$, but $F_1 \wedge F_2 \wedge F_3$ is not satisfiable.

(b) Find four formulas F_0, F_1, F_2, F_3 such that $F_i \wedge F_j \wedge F_k$ is satisfiable for all choices of $i, j, k \in \{0, 1, 2, 3\}$, but $F_0 \wedge F_1 \wedge F_2 \wedge F_3$ is not satisfiable.

Exercise 8: (Switch and and or)
Let $F \equiv G$. Let neither F nor G contain any $\Leftrightarrow$ or $\Rightarrow$. Let F' (respectively G') be the resulting formulas if one changes each $\vee$ in F (respectively G) into $\wedge$ and vice versa. Prove that $F' \equiv G'$.

Send your solutions until Tue 25.10.2022 at 14:00 to your respective tutor.

Please indicate the name of the tutor on your solution sheet. Your solutions have to be in a single file (pdf or similar). Multiple jpeg files (photos) do not count.

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