Dr. D. Frettlöh 25.10.2022

Formal Logic — Exercise Sheet 3

Exercise 9: (Satisfiable vs tautology)
Prove or give a counterexample:

(a) If F is a tautology and $F \Rightarrow G$ is a tautology, then G is a tautology.
(b) If F is satisfiable and $F \Rightarrow G$ is satisfiable, then G is satisfiable.
(c) If F is satisfiable and $F \Rightarrow G$ is a tautology, then G is satisfiable.
(d) If F is satisfiable and $F \Rightarrow G$ is a tautology, then G is a tautology.

Exercise 10: (Horn formula algorithm)
Apply the Marking Algorithm for Horn formulas to the following three formulas F, G and H. Is F (resp. G, resp. H) satisfiable? If yes, please give all valuations $\mathcal{A}$ with $\mathcal{A} \models F$ (resp. $\mathcal{A} \models G$, resp. $\mathcal{A} \models H$).

$$F = (\neg A \vee \neg B \vee \neg D) \wedge \neg E \wedge (\neg C \vee A) \wedge C \wedge B \wedge (\neg G \vee D) \wedge G$$

$$G = (E \wedge C \Rightarrow F) \wedge (B \wedge C \wedge D \Rightarrow A) \wedge (1 \Rightarrow F) \wedge (A \wedge B \wedge E \Rightarrow C) \wedge (B \wedge C \Rightarrow D) \wedge (A \wedge F \Rightarrow B) \wedge (D \Rightarrow 0) \wedge (A \wedge B \Rightarrow E) \wedge (F \Rightarrow A)$$

$$H = (\neg A_1 \vee \neg A_3 \vee \neg A_5 \vee A_6) \wedge (\neg A_2 \vee \neg A_3 \vee A_6) \wedge \neg A_7 \wedge (\neg A_6 \vee \neg A_2 \vee A_7) \wedge A_4 \wedge (A_5 \vee \neg A_4) \wedge (\neg A_4 \vee \neg A_5 \vee A_1) \wedge (\neg A_1 \vee \neg A_2 \vee A_3) \wedge (\neg A_5 \vee \neg A_1 \vee A_2)$$

Exercise 11: (Disney™ Princesses™)
The five™ Disney™ princesses™ Arielle™, Belle™, Cinderella™, Diana™ and Eliza™ are invited to a party™. They state strict opinions:

- Arielle™: If Cinderella™ and Diana™ are coming to the party I will not come.
- Belle™: If Eliza™ is coming I will come as well.
- Cinderella™: If Belle™ and Eliza™ are coming I will come, too.
- Diana™: If Cinderella™ and Eliza™ will come I will come, too.
- Eliza™: I will go to the party anyway.

Translate their statements into a single Horn™ formula F. (Yes, it is possible!) Is F satisfiable? If yes, please give all valuations $\mathcal{A}$ such that $\mathcal{A} \models F$.

Exercise 12: (Easy decisions)
(a) Show that any Horn formula F (in CNF) is satisfiable if each disjunctive clause contains at least one $\neg$.

(b) Show that DNFSAT is in P. That is, let F be in DNF, and let n denote the length of F (that is, the number of symbols). Describe an algorithm that decides in polynomial time with respect to n whether F is satisfiable or not. What is the exact complexity? (in Big O notation with respect to n.)

Send your solutions until Tue 8.11.2022 at 14:00 to your respective tutor.

Please indicate the name of the tutor on your solution sheet. Your solutions have to be in a single file (pdf or similar). Multiple jpeg files (photos) do not count.

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