Formal Logic — Exercise Sheet 4

Exercise 13: (Consumption)
(a) A nonagenarian is asked how he reached this great age. His answer: "A strict diet. For each meal I obey the following rules: Whenever I eat pork I drink beer. When I eat salad I will have pork, too. To each meal I drink beer or maté, or both. If I do not eat tofu then I will have no salad. If I have beer and tofu I will not drink maté. To each meal I will have salad."

Translate this statement into a Horn formula F. (Yes, it is possible. Be creative.) Find all valuations $\mathcal{A}$ with $\mathcal{A} \models F$.

(b) Family Smith plans to make some expensive purchases. However, there are some restrictions. If they buy a motobike then they will not buy a car. If Mrs Smith gets an incentive payment then they will buy a pony for their fastidious daughter. They will buy a washing machine in any case. If they buy a pony and a washing machine then they will need to buy a motobike, too, in order to console their mentally unstable son. If they buy a motobike as well as a pony then they will need to cancel their holiday in Spain.

Translate this statement into a Horn formula, find all valuations $\mathcal{A}$ with $\mathcal{A} \models F$, and show that the Smiths will not spend their holiday in Spain if Mrs Smith gets her incentive payment.

Exercise 14: (Infinitely many formulas)
Find all satisfying valuations for the infinite set of formulas

$$M = \{A_1 \vee A_2, \neg A_2 \vee \neg A_3, A_3 \vee A_4, \neg A_4 \vee \neg A_5, A_5 \vee A_6, \ldots\}$$

(Hint: there are more than two.) Justify that your answer includes all satisfying valuations.

Exercise 15: (Variations of König's Lemma)
(a) Let T be a tree such that each vertex has only finitely many neighbours, and such that T does not contain any infinite path. Show that T has only finitely many vertices.

(b) Show that König's Lemma is wrong if we do not require that each vertex has finitely many neighbours, even the tree contains arbitrary long paths. Hence: construct a counterexample, by finding a tree T that has infinitely many vertices, each level of T is not empty, and T does not contain any infinite path.

Exercise 16: (modus ponendo tollens et resolutio)
(a) Prove the modus ponendo tollens. That is, show that $\{\neg(F \wedge G), F\} \models \neg G$, if F and G are two formulas in propositional logic.

(b) Prove that $\{F \vee L, G \vee \neg L\} \models F \vee G$, if F and G are formulas and L is some literal. (This is Lemma 1.10)

Send your solutions until Tue 15.11.2022 at 14:00 to your respective tutor.

Please indicate the name of the tutor on your solution sheet.
Your solutions have to be in a single file (pdf or similar). Multiple jpeg files (photos) do not count.

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