

Formal Logic — Exercise Sheet 5**Exercise 17: (Resolvents)**

Consider the formulas

$$F = (A \Rightarrow B) \wedge (C \Rightarrow B \vee A) \wedge (A \vee C) \quad \text{and} \quad G = (B \Rightarrow A) \wedge (C \Rightarrow B \vee A) \wedge (A \vee C) \wedge \neg A.$$

Establish the CNF for both formulas and determine $\text{Res}^1(F)$, $\text{Res}^2(F)$, and $\text{Res}^*(F)$ (respectively, $\text{Res}^1(G)$, $\text{Res}^2(G)$, $\text{Res}^*(G)$). Is F satisfiable? Is G satisfiable?

Exercise 18: (Tableaus)

Apply the tableau calculus to the formulas F and G from Exercise 17.

Note that you do not need to establish their CNF. You only need to get rid of the \Rightarrow s first.

Exercise 19: (modus pommel et resolutio)

(a) Prove the modus ponens using the resolution calculus.

(b) Prove the resolution rule again using the resolution calculus. (Sounds weird, but it is OK, since we proved it already in Exercise 16.)

You may try it on your own, but if you do not know how to proceed, here is a hint:

<https://www.math.uni-bielefeld.de/~frettloe/teach/logik/spoilera19.pdf>

Exercise 20: (Efficient resolution calculus)

In the worst case showing the satisfiability of a CNF needs exponentially many steps (for instance, resolutions). But there are several instances where it is efficient. Here are two.

(a) Show that a formula in CNF is always satisfiable if each clause contains at least one negative literal. Is the same true if we replace “negative” with “positive”?

(b) Let F be a formula in CNF containing n different atomic formulas. Let all clauses in F contain at most two literals. Show that, after the resolution algorithm terminated, the number of clauses in $\text{Res}^*(F)$ is at most $O(n^2)$.

(This is the core of the proof that the runtime for 2SAT is $O(n^2)$. That means 2SAT is efficiently decidable.)

Send your solutions until Tue 22.11.2022 at 14:00 to your respective tutor.

Please indicate the name of the tutor on your solution sheet.

Your solutions have to be in a single file (pdf or similar). Multiple jpeg files (photos) do not count.

Jakob Niermann	Tue 16	janiermann@techfak.de
Constantin Lefeld	Tue 16	clefeld@techfak.de
Frederic Alberti	Wed 8	falberti@math.uni-bielefeld.de
Hannah Schweizer	Wed 16	hschweizer@techfak.de
Luigi Esercito / Enrico di Gaspero	Thu 12	lesercito@techfak.de / edigaspero@techfak.de