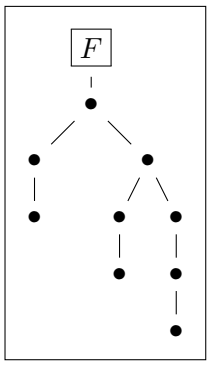
Dr. D. Frettlöh 22.11.2022

Formal Logic — Exercise Sheet 6



Exercise 21: (Find the formula)
Find a formula F that, when the tableau calculus is applied to F, yields the tree in the image on the right. Are there different formulas yielding this tree? For the latter question, we consider formulas as being „not different“ if they arise by just renaming the atomic formulas. This is, $A \vee B$ is not equal to $B \vee A$, nor to $B \vee C$, according to the definition of a formula. But for the purpose of this question we consider them as not being different, since they are equal up to renaming the letters.

Exercise 22: (Relations)
Let B denote the set of all people living in Bielefeld. Which of the following relations are equivalence relations on B, resp. on $\mathbb{Z}$? Give a convincing reason why they are, or provide a counterexample. For the ones that are equivalence relations: list all equivalence classes.

1. $R = \{(a, a) \mid a \in B\}$ on B,
2. $R = \{(a, b) \mid a, b \in B\}$ on B,
3. $R = \{(a, b) \mid a, b \in B, a$ and b live in the same street $\}$ on B,
4. $R = \{(a, b) \mid a, b \in \mathbb{Z}, a - b$ is odd $\}$ on $\mathbb{Z}$,
5. $R = \{(a, b) \mid a, b \in \mathbb{Z}, a - b$ is even $\}$ on $\mathbb{Z}$,
6. $R = \{(a, b) \mid a, b \in \mathbb{Z}, |a - b| \leq 2\}$ on $\mathbb{Z}$.

Exercise 23: (Structures and models)
Consider the following two formulas in first order logic:

$$F = \exists x\, P(x) \wedge \forall y\, Q(a, y), \quad G = \forall x\, \exists y\, \big(P(x, a) \vee Q(x, f(y))\big)$$

List all partial formulas of the formulas, and all terms, and write down the matrix of F. Find a structure that is a model for F, and a structure that is not. Do the same for G.

Exercise 24: (Reflexive, symmetric, transitive)
Consider the formulas

$$F_1 = \forall x\, P(x, x), \quad F_2 = \forall x\, \forall y\, \big(P(x, y) \Rightarrow P(y, x)\big), \quad F_3 = \forall x\, \forall y\, \forall z\, \big((P(x, y) \wedge P(y, z)) \Rightarrow P(x, z)\big).$$

Show that none of these formulas is a consequence of the other two by constructing structures where F_1 and F_2 are true, but F_3 is not; respectively where F_1 and F_3 are true, but F_2 is not; respectively where F_2 and F_3 are true, but F_1 is not.

Send your solutions until Tue 29.11.2022 at 14:00 to your respective tutor.

Please indicate the name of the tutor on your solution sheet.
Your solutions have to be in a single file (pdf or similar). Multiple jpeg files (photos) do not count.

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