

**Formal Logic — Exercise Sheet 7****Exercise 25: (Models and non-models)**

Which of the following structures are models for

$$F = \forall x \exists y P(x, y) \wedge \exists x \neg P(x, x),$$

which are not? Please justify your answers.

- (a)  $U_{\mathcal{A}} = \mathbb{N}, P^{\mathcal{A}} = \{(m, n) \mid m, n \in \mathbb{N}, m \leq n\}$
- (b)  $U_{\mathcal{A}} = \mathbb{N}, P^{\mathcal{A}} = \{(n, n + 1) \mid n \in \mathbb{N}\}$
- (c)  $U_{\mathcal{A}} = \text{Pot}(\mathbb{N})$ , (that is the set of all subsets of  $\mathbb{N}$ ),  $P^{\mathcal{A}} = \{(A, B) \mid A, B \subseteq \mathbb{N}, A \subseteq B\}$
- (d)  $U_{\mathcal{A}} = \{f : \mathbb{R} \rightarrow \mathbb{R} \mid f \text{ differentiable.}\}, P^{\mathcal{A}} = \{(f, g) \mid f = g'\}$

**Exercise 26: (Not a law)**

Show that the two formulas in (a) (respectively, in (b)) are not equivalent to each other by providing a (counter-)example for each.

- (a)  $(\forall x F) \vee (\forall x G) \not\equiv \forall x (F \vee G)$ ,
- (b)  $\exists x (F \wedge G) \not\equiv \exists x F \wedge G$ , where  $x$  is free in  $G$ .

**Exercise 27: (Structures and models II)**

Consider the formulas

$$F = \forall x \forall y \forall z \left( \neg P(x, x) \wedge (P(x, y) \Rightarrow \neg P(y, x)) \wedge (P(x, y) \wedge P(y, z) \Rightarrow P(x, z)) \right).$$

Find two structures  $\mathcal{A}_1, \mathcal{A}'_1$  for  $F$  that are not models for  $F$ , one with a finite universe  $U_{\mathcal{A}_1}$ , one with an infinite universe  $U_{\mathcal{A}'_1}$ .

Moreover, find two models  $\mathcal{A}_2, \mathcal{A}'_2$  for  $F$ , one with a finite universe  $U_{\mathcal{A}_2}$ , one with an infinite universe  $U_{\mathcal{A}'_2}$ .

Please justify why  $\mathcal{A}_1 \not\models F, \mathcal{A}_2 \not\models F, \mathcal{A}'_1 \models F, \mathcal{A}'_2 \models F$ , respectively.

**Exercise 28: (Prenexification)**

Consider the formulas

$$F = \forall x P(x) \wedge \neg \forall y \exists z (P(y) \Rightarrow P(z)), \quad G = \exists x \forall y \forall z ((P(x, y) \vee \neg P(y, z)) \wedge \neg \forall x \exists y (P(x, y) \vee \forall z \neg P(x, z))).$$

Establish the Prenex normal form of  $F$  and  $G$ . Note that it might be necessary to rename some of the variables first.

Send your solutions until Tue 6.12.2022 at 14:00 to your respective tutor.

Please indicate the name of the tutor on your solution sheet.

Your solutions have to be in a single file (pdf or similar). Multiple jpeg files (photos) do not count.

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