DR. D. FRETTLÖH 6.12.2022

Formal Logic — Exercise Sheet 8

Exercise 29: (THE normal form)
Establish THE normal form of

$$F = \exists x\, \forall y\, Q(x,y) \;\Rightarrow\; \exists y\, P(y) \qquad \text{and of}$$

$$G = \neg\exists x\, \forall y\, \big(P(x) \wedge Q(y)\big) \wedge Q(y) \wedge \forall x\, \forall y\, \big(R(x,y) \Rightarrow Q(x)\big)$$

Exercise 30: (Small universes)
(a) Find a formula F of first-order logic without free variables such that F is satisfiable only if $U_{\mathcal{A}}$ has at least three elements. (I.e., F is unsatisfiable for all $U_{\mathcal{A}}$ where $U_{\mathcal{A}}$ has only one or two elements.)

(b) Find a formula F of first-order logic with identity (see Remarks 2.1 and 2.4), and without free variables, such that for all $\mathcal{A}$ with $\mathcal{A} \models F$ holds that $U_{\mathcal{A}}$ has at most two elements.

Exercise 31: (First order consequence)
Prove $\forall x\, (F \Rightarrow G) \models \forall x\, F \Rightarrow \forall x\, G$.

Exercise 32: (The set of all sets)
Consider Russell's paradox (see Wikipedia: Let $U_{\mathcal{A}}$ be the set of all sets. Let $P(x,y)$ be the predicate $x \in y$. What is $\{x \mid \neg P(x,x)\}$? Hence, what is the set of all sets that are not members of themselves?)

Prove that no such set exists by showing that $F = \exists y\, \forall x\, \big(P(x,y) \Leftrightarrow \neg P(x,x)\big)$ is unsatisfiable.

Send your solutions until Tue 13.12.2022 at 14:00 to your respective tutor.

Please indicate the name of the tutor on your solution sheet.
Your solutions have to be in a single file (pdf or similar). Multiple jpeg files (photos) do not count.

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