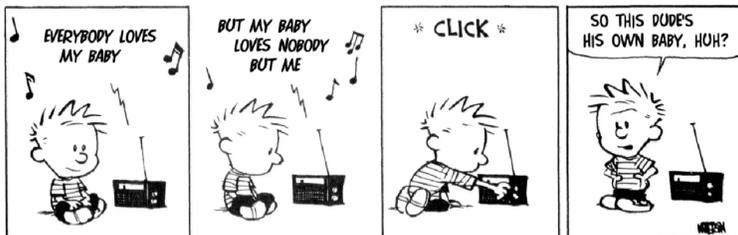## Formal Logic   —   Exercise Sheet 10

Exercise 37: (Dude=Baby)

Consider the cartoon to the right. Derive the same conclusion as the little boy by applying resolution calculus with identity. Choose as universe "all humans". Translate the first two statements into formulas F and G and use two constants:

a for "me" and b for "baby". Show that $a = b$ is a consequence of the two former staments by showing that $F \wedge G \wedge \neg(a = b)$ is unsatisfiable.

Exercise 38: (Smaller infinities?)

Prove that there is no infinite set M with $|M| < |\mathbb{N}|$.

Exercise 39: (Which cardinalities?)

Find the cardinalities of the following sets. Justify your answer (for instance, with a bijection, or the Bernstein-Schröder Theorem, or...)

(a) Herbrand universe $H(F)$ of some formula F in first order logic without constants and without function symbols.

(b) Herbrand universe $H(F)$ of some formula F in first order logic with function symbols.

(c) The set of all sequences $(a_n)_{n \in \mathbb{N}}$ with values $a_n \in \mathbb{N}$.

(d) The set of all sequences $(a_n)_{n \in \mathbb{N}}$ with values $a_n \in \mathbb{R}$.

Exercise 40: (Equal cardinalities)

(a) Show that $|[0, 1]| = \beth_1$.

(b) Show that $|\mathbb{N}^3| = \beth_0$.

Send your solutions until Tuesday 10.1.2023 at 14:00 to your respective tutor.

Please indicate the name of the tutor on your solution sheet.
Your solutions have to be in a single file (pdf or similar). Multiple jpeg files (photos) do not count.

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