

Formal Logic — Exercise Sheet 12**Exercise 45: (Modal logic rules again)**

Prove (resp. disprove) the calculation rules of Exercise 44 again, this time using the tableau calculus for modal logic. Reminder:

Rules 4 and 5 of Theorem 4.1, and showing that $(\diamond F \wedge \diamond G) \equiv \diamond(F \wedge G)$ does not hold in general. *For the latter Remark 3.1 may be helpful.*

Exercise 46: (Tautologies?)

Which of the following four formulas are tautologies, which ones are not? Use the tableau calculus for modal logic to answer this question.

- (a) $H_1 = \Box A \Rightarrow \diamond A$
- (b) $H_2 = \Box A \Rightarrow \Box \Box A$
- (c) $H_3 = \Box \neg A \Rightarrow \neg(\diamond A \wedge \neg B)$
- (d) $H_4 = \neg(\Box(A \Rightarrow B) \wedge \diamond A \wedge \Box \neg B)$

Exercise 47: (According to rank)

Determine the modal rank of the following formulas.

- (a) $F_1 = \Box(A \Rightarrow \Box \diamond \Box B)$
- (b) $F_2 = \Box A \Rightarrow \Box \diamond(\Box \diamond A \wedge \neg B)$
- (c) $F_3 = \Box \neg \diamond(A \vee \neg \diamond \Box(A \vee \Box \diamond \neg B))$
- (d) $F_4 = \Box \diamond \Box(\neg B \wedge \diamond(\diamond \neg \Box B \vee \neg \diamond A) \wedge \Box \diamond A)$

Exercise 48: (Transitivity helps)

Let $F = \Box A \Rightarrow \Box(\Box A \vee B)$.

- (a) Show that F is not a tautology using the tableau calculus for modal logic.
- (b) Prove that F is a tautology if the frame (W, R) is transitive.

Send your solutions until Tuesday 24.1.2023 at 14:00 to your respective tutor.

Please indicate the name of the tutor on your solution sheet.

Your solutions have to be in a single file (pdf or similar). Multiple jpeg files (photos) do not count.

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