Dr. D. Frettlöh 14.10.2025

Formal Logic — Exercise Sheet 1

Exercise 1: (Satisfiable, tautology, equivalent)
Consider the formulas

$$F_1 = (A \wedge \neg B) \vee C, \quad F_2 = A \Rightarrow (B \Rightarrow C), \quad F_3 = (A \Rightarrow B) \Rightarrow C$$

List all partial formulas of F_1. Then decide for each of the three formulas: is the formula a tautology? Is the formula satisfiable? If it is, give a satisfying valuation $\mathcal{A}$ for the respective formula. Are F_1 and F_2 equivalent? Are F_1 and F_3 equivalent? Are F_2 and F_3 equivalent?

Exercise 2: (Trigger sensitive formulas))
(a) Find a formula F containing two atomic formulas A, B with the following property: For every valuation $\mathcal{A}$, changing any of the values of $\mathcal{A}(A)$, $\mathcal{A}(B)$ also changes $\mathcal{A}(F)$.

(b) Find a formula F containing three atomic formulas A, B, C with the following property: For every valuation $\mathcal{A}$, changing any of the values of $\mathcal{A}(A)$, $\mathcal{A}(B)$, $\mathcal{A}(C)$ also changes $\mathcal{A}(F)$.

Exercise 3: (Disney$^{\text{TM}}$ Princesses$^{\text{TM}}$)
(a) The three Disney$^{\text{TM}}$ princesses$^{\text{TM}}$ Arielle$^{\text{TM}}$, Belle$^{\text{TM}}$ and Cinderella$^{\text{TM}}$ are invited to a party. They make the following statements:

Arielle$^{\text{TM}}$:	If Cinderella$^{\text{TM}}$ will not come to the party then I will come to the party.
Belle$^{\text{TM}}$:	If Arielle$^{\text{TM}}$ will come to the party then I will not come.
Cinderella$^{\text{TM}}$:	I will come to the party if Belle$^{\text{TM}}$ and Arielle$^{\text{TM}}$ will come as well; otherwise I will not come.

Translate the statements into one single formula F in propositional logic. Is F satisfiable? If so, give a valuation $\mathcal{A}$ such that $\mathcal{A} \models F$.

(b) Do the same for the following statements.

Arielle$^{\text{TM}}$:	I will not come to the party.
Belle$^{\text{TM}}$:	I will come to the party if Cinderella$^{\text{TM}}$ will not come, and I will not come to the party if Cinderella$^{\text{TM}}$ will come.
Cinderella$^{\text{TM}}$:	I will come to the party if Belle$^{\text{TM}}$ or Arielle$^{\text{TM}}$ (or both) will come as well; otherwise I will not come.

Exercise 4: (Sufficient sets of operators)
(a) Prove that for each formula F there is an equivalent formula G using only $\neg$ and $\Rightarrow$.
(Hint: if you can express $\wedge$ and $\vee$ by combinations of $\neg$ and $\Rightarrow$ then you are done.)

(b) Prove that there is some formula F such that there is no formula G equivalent to F, where G uses only $\wedge$ and $\oplus$. *($\oplus$ means XOR, the exclusive or, see lecture notes).*

(c) Prove that for each formula F there is an equivalent formula G using only $\Rightarrow$ and $\oplus$.

Send your solutions until Tuesday 21.10.2025 at 14:00 to your tutor.

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