Dr. D. Frettlöh 21.10.2025

Formal Logic — Exercise Sheet 2

Exercise 5:
Using propositional logic makes it easy to show identities for sets expressed by union, intersection, and set difference. For instance, $x \in A \cap B$ means: $x \in A \wedge x \in B$, $x \in A \cup B$ means $x \in A \vee x \in B$, $x \notin A$ means $\neg(x \in A)$, $x \in A \setminus B$ means $x \in A \wedge \neg(x \in B)$ and so on. Show the following identities by translating them into formulas and showing their equivalence.

$$C \setminus (A \cap B) = (C \setminus A) \cup (C \setminus B), \quad (A \setminus B) \cap C = A \cap (C \setminus B), \quad A \setminus (B \setminus C) = (A \cap C) \cup (A \setminus B)$$

Look up what a *Venn diagram* is and illustrate the sets by their Venn diagrams.

Exercise 6: (CNF and DNF)
Transform the following formulas into conjunctive normal form and into disjunctive normal form, using Algorithm 1.16 shown in the lecture.

$$F = (A \wedge B \wedge \neg C) \vee (D \wedge \neg E), \qquad G = \neg\Big(A \vee \neg\big(B \wedge (C \vee D)\big)\Big) \wedge (A \Rightarrow B)$$

How many rows would the corresponding truth tables have if you would have used Algorithm 1.19 instead?

Exercise 7: (Equivalence)
(a) Prove the deMorgan laws of Theorem 1.10 using truth tables.

(b) Let us define $\Leftrightarrow$ by $F \Leftrightarrow G := (F \wedge G) \vee (\neg F \wedge \neg G)$ (compare lecture notes page 4, below Example 1.2). Prove $F \Leftrightarrow G \equiv (F \Rightarrow G) \wedge (G \Rightarrow F)$ without using truth tables, only using the calculation rules in Theorem 1.10 and the definitions of $\Leftrightarrow$ and $\Rightarrow$. Please indicate which calculation rules you use and where.

(c) Prove

$$\big(\neg B \wedge \neg(B \wedge A)\big) \wedge \neg\big(C \vee (D \wedge C)\big) \quad \equiv \quad \neg(\neg B \Rightarrow C) \wedge (B \Rightarrow \neg C)$$

without using truth tables, only using the calculation rules in Theorem 1.10. Please indicate which calculation rules you use and where.

Exercise 8: (Borromean formulas)
(a) Find three formulas F_1, F_2, F_3 such that $F_i \wedge F_j$ is satisfiable for all choices of $1 \leq i < j \leq 3$, but $F_1 \wedge F_2 \wedge F_3$ is not satisfiable. Justify your answer (why are the formulas $F_i \wedge F_j$ satisfiable, and why is $F_1 \wedge F_2 \wedge F_3$ not satisfiable?)

(b) Find four formulas F_0, F_1, F_2, F_3 such that $F_i \wedge F_j \wedge F_k$ is satisfiable for all choices of $i, j, k \in \{0, 1, 2, 3\}$, but $F_0 \wedge F_1 \wedge F_2 \wedge F_3$ is not satisfiable. Justify your answer (like in (a)).

Send your solutions until Tuesday 28.10.2025 at 14:00 to your tutor.

Tutor: Lisa Henetmayr lhenetmayr+logik@techfak.de