Dr. D. Frettlöh 28.10.2025

# Formal Logic — Exercise Sheet 3

**Exercise 9:**
Is $\Rightarrow$ associative? Is $\Leftrightarrow$ associative? That is, is $F \Rightarrow (G \Rightarrow H) \equiv (F \Rightarrow G) \Rightarrow H$ true; and/or is $F \Leftrightarrow (G \Leftrightarrow H) \equiv (F \Leftrightarrow G) \Leftrightarrow H$ true? If 'yes', please provide a proof. If 'no', please provide a counterexample.

**Exercise 10: (Horn formula algorithm)**
Apply the Marking Algorithm for Horn formulas to the following three formulas $F, G$ and $H$. Is $F$ (resp. $G$, resp. $H$) satisfiable? If yes, please give all valuations $\mathcal{A}$ with $\mathcal{A} \models F$ (resp. $\mathcal{A} \models G$, resp. $\mathcal{A} \models H$).

$$F = \neg A \wedge (\neg B \vee \neg D) \wedge (\neg C \vee \neg E \vee B) \wedge (\neg D \vee C) \wedge D \wedge (\neg C \vee \neg D \vee E)$$

$$G = (A \wedge B \Rightarrow C) \wedge (B \wedge D \wedge E \Rightarrow C) \wedge (1 \Rightarrow A) \wedge (A \Rightarrow B) \wedge (D \wedge C \Rightarrow E) \wedge (1 \Rightarrow E) \wedge (D \Rightarrow 0)$$

$$H = (\neg A_1 \vee \neg A_2 \vee \neg A_3 \vee A_4) \wedge (\neg A_1 \vee \neg A_3 \vee A_6) \wedge \neg A_6 \wedge A_4 \wedge (\neg A_4 \vee \neg A_5 \vee A_1) \wedge (\neg A_1 \vee \neg A_2 \vee A_3) \wedge (\neg A_5 \vee \neg A_1 \vee A_2) \wedge (A_5 \vee \neg A_4)$$

**Exercise 11: (Disney$^{\text{TM}}$ Princesses$^{\text{TM}}$)**
The five$^{\text{TM}}$ Disney$^{\text{TM}}$ princesses$^{\text{TM}}$ Arielle$^{\text{TM}}$, Belle$^{\text{TM}}$, Cinderella$^{\text{TM}}$, Diana$^{\text{TM}}$ and Eliza$^{\text{TM}}$ are invited to a party$^{\text{TM}}$. Again. They state strict opinions, again:

- Arielle$^{\text{TM}}$: If Cinderella$^{\text{TM}}$ and Diana$^{\text{TM}}$ are coming to the party I will not come.
- Belle$^{\text{TM}}$: If Eliza$^{\text{TM}}$ is coming I will come as well.
- Cinderella$^{\text{TM}}$: If Belle$^{\text{TM}}$ and Eliza$^{\text{TM}}$ are coming I will come, too.
- Diana$^{\text{TM}}$: If Cinderella$^{\text{TM}}$ and Eliza$^{\text{TM}}$ will come I will come, too.
- Eliza$^{\text{TM}}$: I will go to the party anyway.

Translate their statements into a single Horn$^{\text{TM}}$ formula $F$. (Yes, it is possible! Maybe you need some trick.) Is $F$ satisfiable? If yes, please give all valuations $\mathcal{A}$ such that $\mathcal{A} \models F$.

**Exercise 12: (Easy decisions)**
**(a)** Show that any Horn formula $F$ (in CNF) is satisfiable if each disjunctive clause contains at least one $\neg$.

**(b)** State a formula $G$ that does not have an equivalent Horn formula. Justify your answer. (Congratulations if you found one: you just proved that not each formula has an equivalent Horn formula.)

**(c)** Show that DNFSAT is in P. That is, let $F$ be in DNF, and let $n$ denote the length of $F$ (that is, the number of symbols). Describe an algorithm that decides in polynomial time with respect to $n$ whether $F$ is satisfiable or not. What is the exact complexity? (in Big O notation with respect to $n$.)

---

Send your solutions until Tuesday 4.11.2025 at 14:00 to your tutor.

Tutor: Lisa Henetmayr lhenetmayr+logik@techfak.de