Dr. D. Frettlöh 4.11.2025

Formal Logic — Exercise Sheet 4

Aufgabe 13: (Infinite sets of formulas)
Find all satisfying valuations for the following infinite set of formulas.

$$M = \{A_1 \vee A_2, \neg A_2 \vee \neg A_3, A_3 \vee A_4, \neg A_4 \vee \neg A_5, A_5 \vee A_6, \ldots\}$$

Justify your answer. *Hint: there are more than four satisfying valuations.*

Aufgabe 14: (Semantic Consequences)
Let F, G and H be formulas in propositional logic. Show that the following consequences are true. You may use truth tables for at most one of them.

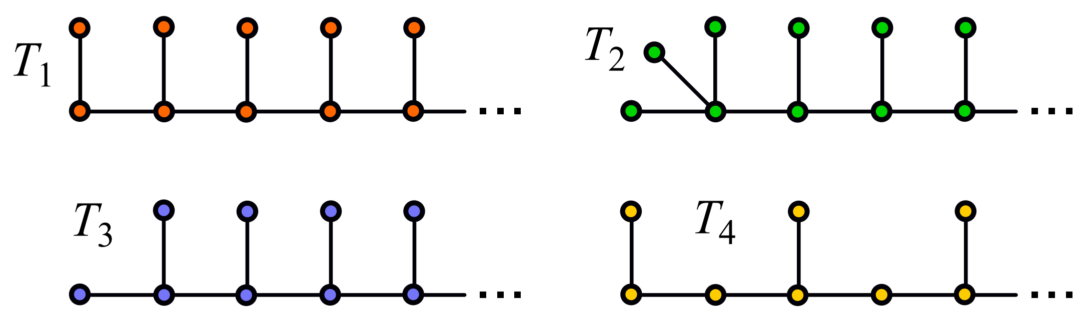
(a) *Modus tollens*: $\{F \Rightarrow G, \neg G\} \models \neg F$.

(b) Resolution: $\{F \vee G, \neg F \vee H\} \models G \vee H$.

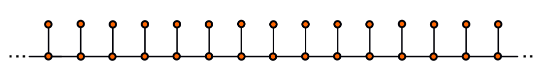
(c) Chain inference: $\{F \Rightarrow G, G \Rightarrow H\} \models F \Rightarrow H$.

Aufgabe 15: (Satisfiable finite sets)
Find a formal proof of Remark 1.28. That is, prove that $\{F_1, \ldots, F_n\}$ is satisfiable if and only if $F = \bigwedge_{i=1}^{n} F_i$ is satisfiable. Is this also true for formulas of the form $F = \bigvee_{i=1}^{n} F_i$?

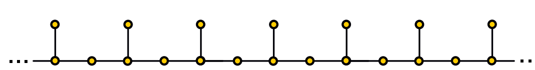
Aufgabe 16: (Infinite trees)
A tree T' is called a subtree of a tree T if the tree T' can be obtained from T by deleting nodes and edges in T. Which of the following infinite trees T_i are subtrees of one of the other T_j? And which are not?



The image is intended to continue in this manner to the right. So, further to the right, T_1, T_2, and T_3 always look like this



and T_4 always looks like this:



Justify your answers, perhaps with a sketch, or a plausible argument. *(The nodes are indistinguishable, i.e., not numbered. Removing a node also removes all edges adjacent to it. Removing an edge does not delete any node.)*

Send your solutions until Tuesday 11.11.2025 at 14:00 to your tutor.

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