Formal Logic — Exercise Sheet 5

Aufgabe 17: (Resolventen)
Transform the following formulas in CNF:

$$F = (C \Rightarrow A \vee B) \wedge (B \Rightarrow A) \wedge \neg A, \qquad G = (C \Rightarrow A \vee B) \wedge \neg\big(B \wedge (C \Rightarrow A)\big) \wedge B \wedge (B \Rightarrow A).$$

Determine $\mathrm{Res}^0(F), \mathrm{Res}^1(F)$ and $\mathrm{Res}^2(F)$. Is F satisfiable?

Apply resolution calculus to the CNF of G. Is G satisfiable?

Exercise 18: (The Dr. Who Diet)
(a) Dr. Who is asked how he reached his remarkable age. His answer: "A strict diet. Whenever I eat, these rules apply: Every meal includes either beans or dates.

If there is alcohol with the meal, there are no dates. Whenever there are dates, I also eat beans. If there are chilies, then I also drink alcohol. Whenever there are beans, there are also chilies and dates."

Translate the statements into a propositional logic formula F, transform F into CNF, and use resolution calculus to decide whether F is satisfiable. (Otherwise, Dr. Who would obviously have been talking nonsense.)

(b) Another time he claims this:

"If there are no beans to eat, I eat chilies. If I drink alcohol with my meal, then I also eat fish fingers. Fish fingers and chilies are always accompanied by beans. Whenever I eat chilies, I also drink alcohol."

Translate these statements into a propositional logic formula G, and show that "Dr. Who eats beans with every meal" is a consequence of G.

Exercise 19: (Forever young...)
Consider the following infinite set of formulas:

$$M = \{A_2 \Rightarrow A_1, A_3 \Rightarrow A_2, A_4 \Rightarrow A_3, A_5 \Rightarrow A_4, A_6 \Rightarrow A_5, \ldots\}.$$

Determine $\mathrm{Res}^1(M)$, $\mathrm{Res}^2(M)$, and in general $\mathrm{Res}^n(M)$. Does the resolution calculus terminate? Justify your answer.

Exercise 20: (Resolution variants)
For each of the following statements, find out whether they are true or false. If "true" give a convincing reason why. If "false" provide a counterexample.

(a) The resolution calculus is still correct if we allow clauses of the form $\{A, \neg A, B, \ldots\}$ to be reduced to $\{B, \ldots\}$.

(b) The resolution calculus is still correct if we allow for the simultaneous resolution of two literals. That means for instance $\{A, B, C\}, \{\neg A, \neg B, C\}$ yields $\{C\}$.

(c) The resolution calculus is still correct if we allow for the simultaneous resolution of two literals wit the same atomic formula. That means for instance $\{A, \neg A, B\}, \{A, \neg A, C\}$ yields $\{B, C\}$.

(d) Clauses of the form $\{A, \neg A, B, \ldots\}$ can just be ignored: omitting them yields the same results.

Send your solutions until Tuesday 18.11.2025 at 14:00 to your tutor.

Tutor: Lisa Henetmayr lhenetmayr+logik@techfak.de