DR. D. FRETTLÖH 18.11.2025

Formal Logic — Exercise Sheet 6

Exercise 21: (The third Dr Who diet)
(a) Show the *modus pollendo tollens* (see lecture notes) using the tableau calculus.

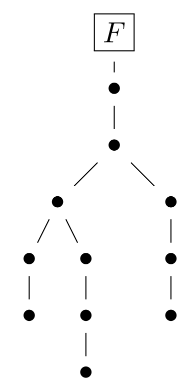
(b) Dr Who is asked how he reached his remarkable age. His answer: "A strict diet. Whenever I eat something the following rules apply: To each meal I will drink alcohol. Each meal I have comes with beans. With each meal I will have carrots, or fish fingers, or both. I will never eat eggs together with carrots. To each meal I will eat either eggs or fish fingers, or both. If I have alcohol to the meal I will have either no beans or no carrots or no fish fingers."

Model the situation into a formula in propositional logic. Use the tableau calculus to show that Dr Who never eats carrots.

Exercise 22: (Find the formula)
Find a formula F in propositional logic such that the tableau calculus applied to F yields the tree on the right. Is there more than one such formula?

For the latter question, we consider formulas to be equal if they can be transformed into each other by renaming the atomic formulas.

That is, $A \vee B$ is not actually the same formula as $B \vee A$, or as $B \vee C$. But for this task, they are considered equal. In contrast, $\neg\neg A \vee B$ is a different formula than $A \vee B$.



Exercise 23: (Serial relations)
A relation R is called **serial**, if for all x there is y such that $(x, y) \in R$ (no dead ends).
(a) Show that if a relation R is symmetric and transitive and serial, then R is reflexive.
(b) Let R be a symmetric and transitive relation. What is wrong with the following proof: "Let $(x, y) \in R$. Because of symmetry we have that $(y, x) \in R$. Because of transitivity it follows that $(x, x) \in R$. Therefore R is reflexive."

Exercise 24: (Relations)
Which of the following relations are equivalence relations? Give a convincing reason why they are, or provide a counterexample. For those relations which are actually equivalence relations: list all equivalence classes.

1. $R = \varnothing$ on $\mathbb{Z}$.
2. $R = \{(a, a) \mid a \in \mathbb{Z}\}$ on $\mathbb{Z}$.
3. $R = \{(a, b) \mid a, b \in \mathbb{Z}, a \leq b\}$ on $\mathbb{Z}$.
4. $R = \{(a, b) \mid a, b \in \mathbb{Z}, \exists\, c \in \mathbb{Z} : a^2 + b^2 = c^2\}$ on $\mathbb{Z}$.
5. $R = \{(a, b) \mid a, b \in \mathbb{Z}, \gcd(a, b) = 2\}$ on $\mathbb{Z}$.
6. $R = \{(a, b) \mid a, b \in \mathbb{Z}, a - b$ is an even number.$\}$

gcd means the greatest common divisor ("größter gemeinsamer Teiler")

Send your solutions until Tuesday 25.11.2025 at 14:00 to your tutor.

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