Formal Logic — Blatt 11

Exercise 41: (According to rank)
Determine the modal rank $\mathrm{MR}(F_i)$ of each of the following formulas.

(a) $F_1 = \diamond(\diamond\Box A \wedge \diamond B)$

(b) $F_2 = \Box A \vee \diamond\neg\Box(\neg A \vee \diamond\Box B)$

(c) $F_3 = \diamond\neg\Box(\Box \diamond (A \vee \neg\Box B) \Rightarrow A)$

(d) $F_4 = \diamond\Box\neg \diamond \big(\neg \diamond B \wedge \Box \diamond (\Box\neg \diamond B \vee \neg\Box A) \wedge \diamond \diamond \Box A\big)$

Exercise 42: (Tautologies?)
Which of the following formulas are tautologies, which are not? Use the tableau calculus to solve this exercise. Recall that a formula is a tautology if and only if its negation is unsatisfiable.

(a) $F_1 = \Box A \Rightarrow \diamond A$
(b) $F_2 = \diamond A \Rightarrow A$
(c) $F_3 = \Box\neg A \Rightarrow \neg(\diamond A \wedge \neg B)$
(d) $F_4 = \neg(\Box(A \Rightarrow B) \wedge \diamond A \wedge \Box\neg B)$

Exercise 43: (More rules)
Use the tableau calculus for this exercise.

(a) Prove Rule 4 of Theorem 3.7 by showing that $\Box(F \Rightarrow G) \Rightarrow (\diamond F \Rightarrow \diamond G)$ is a tautology.
(b) Prove Rule 7 of Theorem 3.7 by showing that $\diamond(F \wedge G) \Leftrightarrow \diamond F \wedge \diamond G$ is a tautology. First eliminate the $\Leftrightarrow$ by transforming the formula into some equivalent formula that uses only the symbols $\neg, \wedge, \vee, \Box, \diamond$.

Exercise 44: (Dr. Who and the infernal device)
Dr Who encounters an infernal device that might possibly explode, thus destroying all life, time, universes and so on. It has three levers marked A, B and C. All three can be shifted either up or down, but the levers interact with each other. For instance, it is always true that not both of lever A and lever B can be up. It is also always true that if lever C is up then lever A is up or lever B is down (or both). At each time holds that if lever C is up then lever B is up, too.

The infernal device will explode if lever C is up. Translate the situation above in formulas F_2, F_2, F_3 in modal logic and use the tableau calculus to show that the infernal device will never explode. Hence, show that $F_1 \wedge F_2 \wedge F_3 \wedge \diamond C$ is unsatisfiable.

Send your solutions until Tuesday 13.1.2026 at 14:00 to your tutor.

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