Formal Logic — Blatt 12

Exercise 45: (Transitivity helps)
Let $F = \Box A \Rightarrow \Box(\Box A \vee B)$.

(a) Show that F is not a tautology using the tableau calculus for modal logic.
(b) Prove that F is a tautology if we restrict ourselves to transitive frames (W, R) only.

Exercise 46: (Even more tautologies)
Use the tableau calculus for (a). Justify your answers for (b) and (c).

(a) Show that $F = G \Rightarrow \diamond G$ is satisfiable, but not a tautology.
(b) If we require the frame (W, R) for F from (a) to be reflexive, is F a tautology?
(c) If we require the frame (W, R) for F from (a) to be transitive, is F a tautology?

Exercise 47: (Cardinalities)
Determine the cardinality of each of the following sets and prove your answer. You may describe the bijections you use in any way, using `if ... then`, or enumerated lists, or functions, or ... You may as well use the Schröder-Bernstein Theorem.

(a) $\mathcal{P}(\mathbb{R})$

(b) $\mathbb{Q} \times \mathbb{Q}$

(c) The set of all sequences $(a_n)_{n\in\mathbb{N}}$ with values $a_n \in \mathbb{R}$.

(d) The set of all functions $f : \mathbb{R} \to \mathbb{R}$.

Exercise 48: Smaller infinities?
Show that there are no infinite sets M such that $|M| < \beth_0$.

Send your solutions until Tuesday 20.1.2026 at 14:00 to your tutor.

Tutor: Lisa Henetmayr lhenetmayr+logik@techfak.de