Formal Logic — Exercise Sheet 2

Exercise 5: (Equivalence)
(a) Prove the deMorgan laws of Theorem 1.10 using truth tables.

(b) Let us define $\Leftrightarrow$ by $F \Leftrightarrow G := (F \wedge G) \vee (\neg F \wedge \neg G)$ (compare lecture notes page 4, below Example 1.2). Prove $F \Leftrightarrow G \equiv (F \Rightarrow G) \wedge (G \Rightarrow F)$ without using truth tables, only using the calculation rules in Theorem 1.10 and the definitions of $\Leftrightarrow$ and $\Rightarrow$. Please indicate which calculation rules you use and where.

(c) In the same manner, prove $(F \Leftrightarrow G) \wedge \neg(G \Rightarrow H) \equiv \neg(F \wedge G \Rightarrow H)$.

Exercise 6: (To be commutative, or not to be commutative)
For each of the following four statements, find a proof or give a counterexample.

(a) $\Rightarrow$ is commutative.

(b) $\Leftrightarrow$ is commutative.

(c) $\Rightarrow$ is associative.

(d) $\Leftrightarrow$ is associative.

Exercise 7: (Satisfiable vs tautology)
For each of the following four statements, find a proof or give a counterexample.

(a) If F is a tautology and $F \Rightarrow G$ is a tautology, then G is a tautology.

(b) If F is satisfiable and $F \Rightarrow G$ is satisfiable, then G is satisfiable.

(c) If F is satisfiable and $F \Rightarrow G$ is a tautology, then G is satisfiable.

(d) If F is satisfiable and $F \Rightarrow G$ is a tautology, then G is a tautology.

Exercise 8: (Borromean formulas)
(a) Find three formulas F_1, F_2, F_3 such that $F_i \wedge F_j$ is satisfiable for all choices of $1 \leqslant i < j \leqslant 3$, but $F_1 \wedge F_2 \wedge F_3$ is not satisfiable.

(b) Find four formulas F_0, F_1, F_2, F_3 such that $F_i \wedge F_j \wedge F_k$ is satisfiable for all possible choices of $i, j, k \in \{0, 1, 2, 3\}$, but $F_0 \wedge F_1 \wedge F_2 \wedge F_3$ is not satisfiable.

Send your solutions until Tuesday 27.10.2026 at 14:00 to your tutor.

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