Dr. D. Frettlöh 27.10.2026

# Formal Logic — Exercise Sheet 3

**Exercise 9: (CNF and DNF)**
Transform the following formulas into conjunctive normal form and into disjunctive normal form, using Algorithm 1.16 shown in the lecture.

$$F = \neg\Big(A \vee \neg\big(B \wedge \neg(C \vee D)\big)\Big), \qquad G = \neg(A \Rightarrow B) \vee (\neg C \wedge D \wedge \neg E).$$

How many rows would the corresponding truth tables have if you would have used Algorithm 1.19 instead?

**Exercise 10: (Horn formula algorithm)**
Apply the Marking Algorithm for Horn formulas to the following three formulas $F, G$ and $H$. Is $F$ (resp. $G$, resp. $H$) satisfiable? If yes, please give all valuations $\mathcal{A}$ with $\mathcal{A} \models F$ (resp. $\mathcal{A} \models G$, resp. $\mathcal{A} \models H$).

$$F = \neg A \wedge (\neg B \vee \neg D) \wedge (\neg C \vee \neg E \vee B) \wedge (\neg D \vee C) \wedge D \wedge (\neg C \vee \neg D \vee E)$$

$$G = (A \wedge B \Rightarrow C) \wedge (B \wedge D \wedge E \Rightarrow C) \wedge (1 \Rightarrow A) \wedge (A \Rightarrow B) \wedge (D \wedge C \Rightarrow E) \wedge (1 \Rightarrow E) \wedge (D \Rightarrow 0)$$

$$H = (\neg A_1 \vee \neg A_2 \vee \neg A_3 \vee A_4) \wedge (\neg A_1 \vee \neg A_3 \vee A_6) \wedge \neg A_6 \wedge A_4 \wedge (\neg A_4 \vee \neg A_5 \vee A_1) \wedge (\neg A_1 \vee \neg A_2 \vee A_3) \wedge (\neg A_5 \vee \neg A_1 \vee A_2) \wedge (A_5 \vee \neg A_4)$$

**Exercise 11: (Disney™ Princesses™)**
**(a)** The four™ Disney™ princesses™ Arielle™, Belle™, Cinderella™, and Diana™ are invited to a party™. They state strict opinions:

- Arielle™: If Cinderella™ and Diana™ are coming to the party I will not come.
- Belle™: If Cinderella™ is coming I will come as well.
- Cinderella™: If Arielle™ will come to the party I will not come.
- Diana™: Cinderella™ and me will both come to the party.

Translate their statements into a single Horn™ formula $F$. Is $F$ satisfiable? If yes, please give all valuations $\mathcal{A}$ such that $\mathcal{A} \models F$.

**(b)** Later, the five™ Disney™ princesses™ Arielle™, Belle™, Cinderella™, Diana™ and Eliza™ are invited to a party™. Again. They state strict opinions, again.

- Arielle™: If Cinderella™ and Diana™ are coming to the party I will not come.
- Belle™: If Eliza™ is coming I will come as well.
- Cinderella™: If Belle™ and Eliza™ are coming I will come, too.
- Diana™: If Cinderella™ and Eliza™ will come I will come, too.
- Eliza™: I will go to the party anyway.

Again, translate their statements into a single Horn™ formula $G$.(Yes, it is possible! Maybe you need some trick.) Is $G$ satisfiable? If yes, please give all valuations $\mathcal{A}$ such that $\mathcal{A} \models G$.

**Exercise 12: (Easy decisions)**
**(a)** Show that any Horn formula $F$ (in CNF) is satisfiable if each disjunctive clause contains at least one $\neg$.

**(b)** State a formula $G$ that does not have an equivalent Horn formula. Justify your answer. (Congratulations if you found one: you just proved that not each formula has an equivalent Horn formula.)

**(c)** Show that DNFSAT is in P. That is, let $F$ be in DNF, and let $n$ denote the length of $F$ (that is, the number of symbols). Describe an algorithm that decides in polynomial time with respect to $n$ whether $F$ is satisfiable or not. What is the exact run time complexity? (in Big O notation with respect to $n$.) You may use some additional memory, like $O(n)$ or less.

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Send your solutions until Tuesday 3.11.2026 at 14:00 to your tutor.

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