Formal Logic — Exercise Sheet 4

Exercise 13: (modus ponendo tollens et resolutio)
(a) Prove the modus ponendo tollens. That is, show that $\{\neg(F \wedge G), F\} \models \neg G$, if F and G are two formulas in propositional logic.

(b) Prove that $\{F \vee L, G \vee \neg L\} \models F \vee G$, if F and G are formulas and L is some literal.

Exercise 14: (Variations of König's Lemma)
(a) Let T be a tree such that each vertex has only finitely many neighbours, and such that T does not contain any infinite path. Show that T has only finitely many vertices.

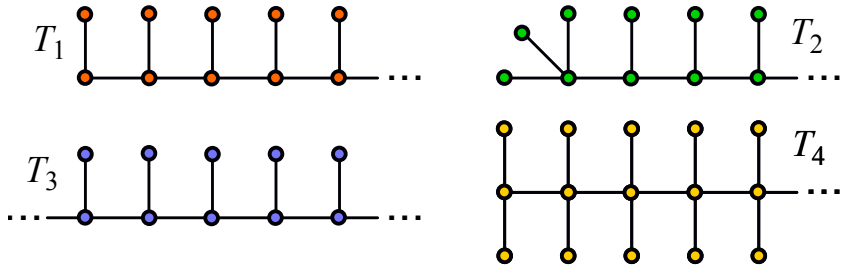
(b) Show that König's Lemma is wrong if we do not require that each vertex has finitely many neighbours, even the tree contains arbitrary long paths. Hence: construct a counterexample, by finding a tree T that has infinitely many vertices, each level of T is not empty (that is, for each $n \in \mathbb{N}$ there is a path of length n starting from the root), and T does not contain any infinite path.

Exercise 15: (Infinitely many formulas)
Find all valuations for $A_1, A_2, \ldots$ satisfying the infinite set of formulas

$$M = \{A_2 \Rightarrow A_1, A_3 \Rightarrow A_2, A_4 \Rightarrow A_3, A_5 \Rightarrow A_4, \ldots\}$$

(Hint: there are more than one.) Justify why your answer includes all satisfying valuations.

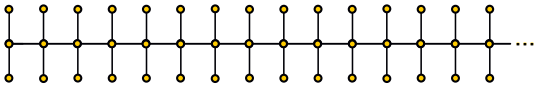
Aufgabe 16: (Infinite trees)
A tree T' is called a subtree of a tree T if the tree T' can be obtained from T by deleting nodes and edges in T. Which of the following infinite trees T_i are subtrees of one of the other T_j? And which are not?



The image is intended to continue in this manner to the right. So, further to the right (and, in case of T_3, also to the left), T_1, T_2, and T_3 always look like this



and T_4 always looks like this:



Justify your answers, perhaps with a sketch, or a plausible argument. *(The nodes are indistinguishable, i.e., not numbered. Removing a node also removes all edges adjacent to it. Removing an edge does not delete any node.)*

Send your solutions until Tuesday 10.11.2026 at 14:00 to your tutor.

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