Simple dynamical models interpreting climate data and their meta-stability

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Abstract

Simple models of the earth's energy balance are able to interpret some qualitative aspects of the dynamics of paleo-climatic data. In the 1980s this led to the investigation of periodically forced dynamical systems of the reaction-diffusion type with small Gaussian noise, and a rough explanation of glacial cycles by Gaussian meta-stability. A spectral analysis of Greenland ice time series performed at the end of the 1990s representing average temperatures during the last ice age suggest an $\alpha-$stable noise component with an $\alpha \sim 1.75$. Based on this observation, papers in the physics literature attempted an interpretation featuring dynamical systems perturbed by small Lévy noise. We study exit and transition between meta-stable states for solutions of stochastic differential equations and stochastic reaction-diffusion equations derived from this prototype.

Interpreting paleo-climatic time series by simple dynamical systems with noise leads to statistical model selection problems. For instance, one needs an efficient testing method for the best fitting α-stable noise component. We develop a statistical testing method based on the p-variation of the solution trajectories of SDE with Lévy noise, for example by showing asymptotic normality or asymptotic β-stability of their approximations along finite interval partitions.

It has been suggested that the exit and transition characteristics of dynamical systems perturbed by small Lévy noise approach Gaussian behavior as the heavy tails of their jump laws become exponentially light of order γ, i.e. if for $x \to \infty$ they are given by $\exp(-cx^{\gamma})$, and as $\gamma \to 2$. We show that this is surprisingly false, by exhibiting an intriguing phase transition at $\gamma = 1$.