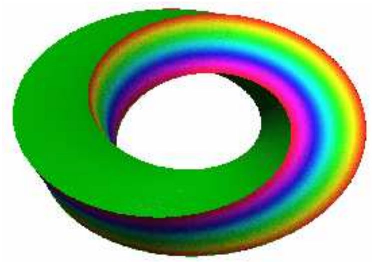
Universität Bielefeld

Fakultät für Mathematik



Second Workshop on Random Dynamical Systems

17 – 19 November 2008

Department of Mathematics

University of Bielefeld

Room H10

This workshop is part of the conference program of the DFG-funded CRC 701

Spectral Structures and Topological Methods in Mathematics

at the University of Bielefeld

Organizer: Barbara Gentz (gentz@math.uni-bielefeld.de)

http://www.math.uni-bielefeld.de/~gentz/pages/WS08/Workshop_RDS08/RDS08.html

Programme

Monday, 17 November 2008

9:00 – 9:30	*Registration*
9:30 – 9:35	*Welcome*
9:35 – 10:25	**Anton Bovier** (Rheinische Friedrich-Wilhelms-Universität Bonn) Kawasaki dynamics in large volumes
10:35 – 11:25	**Alessandra Bianchi** (WIAS Berlin) Coupling in potential wells: from average to pointwise estimates of metastable times
11:25 – 12:00	*Coffee break*
12:00 – 12:50	**Gabriel Lord** (Heriot-Watt University, Edinburgh) Stochastic travelling waves in neural tissue
12:50 – 15:30	*Lunch break*
15:30 – 16:20	**Dirk Blömker** (Universität Augsburg) Stabilization due to additive noise
16:20 – 17:00	*Coffee break*
17:00 – 17:50	**Gary Froyland** (University of New South Wales, Sydney) Coherent sets and isolated spectrum for random Perron–Frobenius cocycles

Tuesday, 18 November 2008

9:00 – 9:50	**Denis Talay** (INRIA, Sophia Antipolis) On Lagrangian McKean–Vlasov particle systems with interactions governed by conditional expectations
10:00 – 10:50	**Peter Kloeden** (Goethe-Universität Frankfurt a. M.) The numerical approximation of stochastic PDEs
10:50 – 11:30	*Coffee break*
11:30 – 12:20	**Peter Imkeller** (HU Berlin) Simple dynamical models interpreting climate data and their metastability
12:20 – 15:00	*Lunch break*
15:00 – 15:50	**Nils Berglund** (MAPMO–CNRS, Orléans) Metastability in systems with bifurcations
15:50 – 16:30	*Coffee break*
16:30 – 17:20	**Michael Allman** (Warwick Mathematics Institute) Breaking the chain
17:30 – 18:20	**Yuri Kifer** (Hebrew University, Jerusalem) From PET to SPLIT
19:30 –	*Joint dinner in the city* Weinbar 3A, Oberntorwall 3a, 33615 Bielefeld (Please note: For the dinner, prior registration is required.)

Wednesday, 19 November 2008

9:00 – 9:50	**Jiří Černý** (ETH Zürich) Convergence to fractional kinetics for some models on $\mathbb{Z}^d$
10:00 – 10:50	**Leonid Bunimovich** (Georgia Institute of Technology, Atlanta, GA) Where to place hole to achieve the fastest escape
10:50 – 11:30	*Coffee break*
11:30 – 12:20	**Luc Rey-Bellet** (University of Massachusetts, Amherst, MA) Large deviations for hyperbolic billiards and non-uniformly hyperbolic dynamical systems
12:20 – 14:30	*Lunch and coffee break*
14:30 – 15:20	**Peter Reimann** (Universität Bielefeld) Enhanced diffusion in a tilted periodic potential: universality, scaling, and the effect of disorder
15:30 – 16:20	**Michael Zaks** (HU Berlin) Globally coupled excitable systems: attempt of non-Markovian description
16:20 – 16:30	*Closing of the workshop*

Abstracts

Michael Allman (Warwick Mathematics Institute)

Breaking the chain

We consider the motion of a Brownian particle in $\mathbb{R}$, moving between a particle fixed at the origin and another moving deterministically away at slow speed $\epsilon > 0$. The middle particle interacts with its neighbours via a potential of finite range $b > 0$, with a unique minimum at $a > b/2$. We say that the chain of particles breaks on the left- or right-hand side when the middle particle is greater than a distance b from its left or right neighbour, respectively. We study the asymptotic location of the first break of the chain in the limit of small noise, in the case where $\epsilon = \epsilon(\sigma)$ and $\sigma > 0$ is the noise intensity.

Nils Berglund (MAPMO–CNRS, Orléans)

Metastability in systems with bifurcations

The overdamped motion of a Brownian particle in a multiwell potential is metastable for weak noise: Transitions between potential minima take place on exponentially long time scales, which are usually governed by the classical Eyring–Kramers law. This law, however, breaks down when the potential landscape undergoes bifurcations, and some saddles become flat. We will show how recent results by Bovier, Eckhoff, Gayrard and Klein, yielding the first mathematically rigorous proof of the Eyring–Kramers formula, can be extended to cases with flat saddles. We will also present applications to stochastic PDEs.

Joint work with Barbara Gentz (Bielefeld).

Alessandra Bianchi (WIAS Berlin)

Coupling in potential wells: from average to pointwise estimates of metastable times

In many situations of interest, the potential theoretic approach to metastability allows to derive sharp estimates for quantities characterizing the metastable behavior of a given system. In this framework, the average metastable times can be expressed through the capacity of corresponding metastable sets, and capacities can be estimated with the application of two different variational principles, providing upper and lower bounds. After recalling these basic concepts and techniques, I will describe a new method to couple the dynamics inside potentials wells. Under some general hypothesis, I will show that this yields sharp estimates on metastable times, pointwise on any metastable set. Our key example will the random field Curie–Weiss model.

Joint work with A. Bovier (Bonn) and D. Ioffe (Haifa).

Dirk Blömker (Universität Augsburg)

Stabilization due to additive noise

We present results on stabilization of solutions to semilinear parabolic PDEs near a change of stability due to additive degenerate noise. Our analysis is based on the rigorous derivation of a stochastic amplitude equation for the dominant Fourier mode and on careful estimates on its solution. Furthermore, a few numerical examples which corroborate our theoretical findings are presented. Amplitude equations are derived via a multi-scale analysis based on the natural separation of time-scales near a change of stability.

Joint work with M. Hairer (Warwick) and G. Pavliotis (Imperial).

Anton Bovier (Rheinische Friedrich-Wilhems-Universität Bonn)

Kawasaki dynamics in large volumes

In this talk we present results on metastability in large volumes at low temperatures for conservative (Kawasaki) dynamics of on Ising lattice gas. Let β denote the inverse temperature and let $\Lambda_\beta \subset \mathbb{Z}^2$ be a square box with periodic boundary conditions such that $\lim_{\beta\to\infty} |\Lambda_\beta| = \infty$. We run the dynamics on Λ_β starting from a random initial configuration where all the droplets are small. For large β, and for interaction parameters that correspond to the metastable regime, we investigate how the transition from the metastable state (with only small droplets) to the stable state (with one or more large droplets) takes place under the dynamics. This transition is triggered by the appearance of a single *critical droplet* somewhere in Λ_β. Using potential-theoretic methods, we compute the *average nucleation time* (= the first time a critical droplet appears and starts growing) up t o a multiplicative factor that tends to one as $\beta \to \infty$. It turns out that this time grows as $K\beta e^{\Gamma\beta}/|\Lambda_\beta|$ for Kawasaki dynamics, where Γ is the local grand-canonical energy to create a critical droplet and K is a constant reflecting the geometry of the critical droplet. The fact that the average nucleation time is inversely proportional to $|\Lambda_\beta|$ is referred to as *homogeneous nucleation*, because it says that the critical droplet for the transition appears essentially independently in small boxes that partition Λ_β.

Joint work with Frank den Hollander and Cristian Spitoni.

Leonid Bunimovich (Georgia Institute of Technology, Atlanta, GA)

Where to place hole to achieve the fastest escape

The question in the title seems has been overlooked in the theory of open dynamical systems. Choose in a phase space of a measure preserving dynamical system two subsets A and B of a positive measure. Consider now two open dynamical systems with holes A and B respectively. In which one a survival probability will be smaller? (One will immediately think about a size (measure) of a hole. However, the situation is much more complex.) This question can be completely answered for some classes of dynamical systems. Moreover, the corresponding results hold for all (finite!) times starting with some exactly defined moment (rather than for "sufficiently large" times or for intervals of time with ends described by some functions of some "small" parameter). A variety of new problems (including the ones on small random perturbations) arise in this area. Overall, size matters but dynamics can matter even more.

Jiří Černý (ETH Zürich)

Convergence to fractional kinetics for some models on $\mathbb{Z}^d$

We show that the random walk among unbounded random conductances and Metropolis dynamics for trap model on $\mathbb{Z}^d$, $d \geq 3$, converge after rescaling to fractional kinetics process. Such convergence is equivalent to aging in these models.

Gary Froyland (University of New South Wales, Sydney)

Coherent sets and isolated spectrum for random Perron–Forbenius cocycles

Transport and mixing processes play an important role in many natural phenomena. Ergodic theoretic approaches to identifying slowly mixing structures in dynamical systems have been developed around the Perron–Frobenius operator and its eigenfunctions. We describe an extension of these techniques to random dynamical systems in which one can observe random slowly dispersive structures, which we term *coherent sets*. We study cocycles generated by expanding interval maps and the rates of decay for functions of bounded variation under the action of the associated Perron–Frobenius cocycles. We show that when the generators are piecewise affine and share a common Markov partition, the Lyapunov spectrum of the Perron–Frobenius cocycle has at most finitely many isolated points. We also state a strengthened version the Multiplicative Ergodic Theorem for random products of non-invertible matrices, and develop a numerical algorithm to approximate the Oseledets subspaces that describe coherent sets.

Peter Imkeller (HU Berlin)

Simple dynamical models interpreting climate data and their meta-stability

Simple models of the earth's energy balance are able to interpret some qualitative aspects of the dynamics of paleo-climatic data. In the 1980s this led to the investigation of periodically forced dynamical systems of the reaction–diffusion type with small Gaussian noise, and a rough explanation of glacial cycles by Gaussian meta-stability. A spectral analysis of Greenland ice time series performed at the end of the 1990s representing average temperatures during the last ice age suggest an α-stable noise component with an $\alpha \sim 1.75$. Based on this observation, papers in the physics literature attempted an interpretation featuring dynamical systems pertubed by small Lévy noise. We study exist and transition between meta-stable states for solutions of stochastic differential equations and stochastic reaction–diffusion equations derived from this prototype.

Interpreting paleo-climatic time series by simple dynamical systems with noise leads to statistical model selection problems. For instance, one needs an efficient testing method for the best fitting α-stable noise component. We develop a statistical testing method based on the p-variation of the solution trajectories of SDE with Lévy noise, for example by showing asymptotic normality or asymptotic β-stability of their approximations along finite interval partitions.

It has been suggested that the exit and transition characteristics of dynamical systems pertubed by small Lévy noise approach Gaussian behavior as the heavy tails of their jump laws become exponentially light of order γ, i.e. if for $x \to \infty$ they are given by $\exp(-cx^\gamma)$, and as $\gamma \to 2$. We show that this is surprisingly false, by exhibiting an intriguing phase at $\gamma = 1$.

Joint work with Claudia Hein, Michael Högele, Ilya Pavlyukevich and Torsten Wetzel.

Yuri Kifer (Hebrew University, Jerusalem)

From PET to SPLIT

Various forms of the polynomial ergodic theorem (PET) which attracted substantial attention in ergodic theory study the limits of expressions having the form $1/N \sum_{n=1}^{N} T^{q_1(n)} f_1 \cdots T^{q_\ell(n)} f_\ell$ where T is a weakly mixing measure preserving transformation, f_i's are bounded measurable functions and q_i's are polynomials taking on integer values on the integers. Motivated partially by these results we obtain a central limit theorem for expressions of the form

$$1/\sqrt{N} \sum_{n=1}^{N} (X_1(q_1(n)) X_2(q_2(n)) \cdots X_\ell(q_\ell(n)) - a_1 a_2 \ldots a_\ell)$$

(sum-product limit theorem-SPLIT) where X_i's are fast α-mixing bounded stationary processes, $a_j = EX_j(0)$ and q_i's are positive functions taking on integer values on integers with some growth conditions which are satisfied, for instance, when q_i's are polynomials of growing degrees. This result can be applied to the case when $X_i(n) = T^n f_i$ where T is a mixing subshift of finite type, a hyberbolic diffeomorphism or an expanding transformation taken with a Gibbs invariant measure, as well, as to the case when $X_i(n) = f_i(\xi_n)$ where ξ_n is a Markov chain satisfying the Doeblin condition considered as a stationary process with respect to its invariant measure.

Peter Kloeden (Goethe-Universität Frankfurt a.M.)

The numerical approximation of stochastic PDEs

We consider parabolic stochastic partial differential equations (SPDE) driven by additive space–time white noise. A result of Davie & Gaines (2000) shows that the overall computational order for the strong convergence of numerical schemes using only increments of the noise cannot exceed 1/6. We introduce a new numerical scheme for the time discretization of the finite dimensional Galerkin SDEs, which we call the exponential Euler scheme, and show that its computational order is 1/3. Our scheme takes advantage of the smoothening effect of the Laplace operator and of a linear functional of the noise. This talk is based on the paper A. Jentzen and P.E. Kloeden, Overcoming the order barrier in the numerical approximation of SPDEs with additive space–time noise, *Proc. Roy. Soc. London* (to appear).

Gabriel Lord (Heriot-Watt University, Edinburgh)

Stochastic travelling waves in neural tissue

We start by discussing the computation of travelling in the presence of stochastic forcing (both Ito and Stratonovich). We will compare levelset computations of wavespeeds to computation through a minimization of the L^2 norm to a reference function and examine the effect of spatial correlation. We apply these techniques to models of neural tissue and in icular the Nagumo equation and the Baer–Rinzel model for wave propagation in dendrites. We show that the dendritic tree can act as a filter and robustness to noise.

Peter Reimann (Universität Bielefeld)

Enhanced diffusion in a tilted periodic potential: universality, scaling, and the effect of disorder

The diffusion of an overdamped Brownian particle in a tilted periodic potential may exhibit a pronounced enhancement over the free thermal diffusion in a small vicinity of the so-called critical tilt, i.e. the threshold bias at which deterministic running solutions set in. Weak disorder in the form of small, time-independent deviations from a strictly spatially periodic potential may further boost this diffusion peak by orders of magnitude. The theoretical predictions are in good agreement with experimental observations.

Luc Rey-Bellet (University of Massachusetts, Amherst, MA)

Large deviations for hyberbolic billiards and non-uniformly hyperbolic dynamical systems

We present large deviation results for ergodic averages dynamical systems which are chaotic and admit a SRB measure but are not uniformly hyperbolic. For example our results cover the Sinai billiard (Lorentz Gas) as well as Henon maps and other nonuniformly hyperbolic dynamical systems. The analysis is based on a symbolic representation of these dynamical systems introduced by L.S. Young (the so-called Young towers). We also discuss some applications to steady states in nonequilibrium statistical mechanics and to fluctuations of entropy production in such systems.

This a joint work with Lai-Sang Young (Courant Institute, NYU).

Denis Talay (INRIA, Sophia Antipolis)

On Lagrangian McKean–Vlasov particle systems with interactions governed by conditional expectations

Joint work with Mireille Bossy and Jean-François Jabir.

Michael Zaks (HU Berlin)

Globally coupled excitable systems: attempt of non-Markovian description

In a network of stochastic excitable units with three discrete states, we characterize each state by the waiting time density function. The limit of large ensemble yields the non-Markovian mean field equations: nonlinear integral equations for the populations of three states. In the framework of those equations, different instabilities of steady solutions are discussed. Results are compared with simulations of discrete units and of coupled FitzHugh–Nagumo systems.

Joint work with L. Schimansky-Geier and H. Leonhardt (Berlin).

Registered participants

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Florent Barret	(CMAP, Palaiseau)
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Dirk Blömker	(Universität Augsburg)
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Giacomo di Gesù	(Berlin Mathematical School)
Ale Jan Homburg	(University of Amsterdam)
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(as of 13 November 2008)