COMPUTING STOHASTIC PHASE-FIELD MODELS

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ABSTRACT

Our main goal is the (correct) numerical approximation of stochastic phase-fields such as, the Allen-Cahn problem with additive white noise in one-dimensional space:

$$\partial_t u(x,t) - \Delta u(x,t) + \frac{1}{\epsilon^2}(u(x,t)^3 - u(x,t)) = \epsilon^\gamma \partial_{xt} W(x,t), \quad x \in (-1,1) \text{ and } t \in (0,T],$$

with Neumann boundary conditions and $u(x,0) = u_0(x)$. We then have to relate our numerical results to theoretical results from the probabilistic analysis of scaling limits conducted mainly by Funaki and Brassesco, De Masi & Presutti in the 1990's [2, 1].

From a compuational view-point, the main difficulty for a rigorous numerical discretization of this SPDE, which is the baby-version of more complicated phase-field systems, is the presence of the time-space white noise as a forcing term [6, 7, 5].

We conduct a finite element discretization in two stages: (1) we regularize the white noise and study the regularized problem, and (2) we approximate the regularized problem and derive a finite element Monte Carlo simulation scheme [3].

In this talk, I will review recent advances on the topic where we analyze the interface behavior, as its thickness ϵ stays finite. Asymptotic analysis shows that the stochastic solution and its approximation remain "close" to a set of functions where the interface makes sense. This is particularly useful to relate numerical results to theory and it has the potential to define stochastic diffuse interfaces in dimensions higher than one [4].

(*Based on joint work with Markos Katsoulakis, Giorgos Kossioris and Marco Romito*)

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