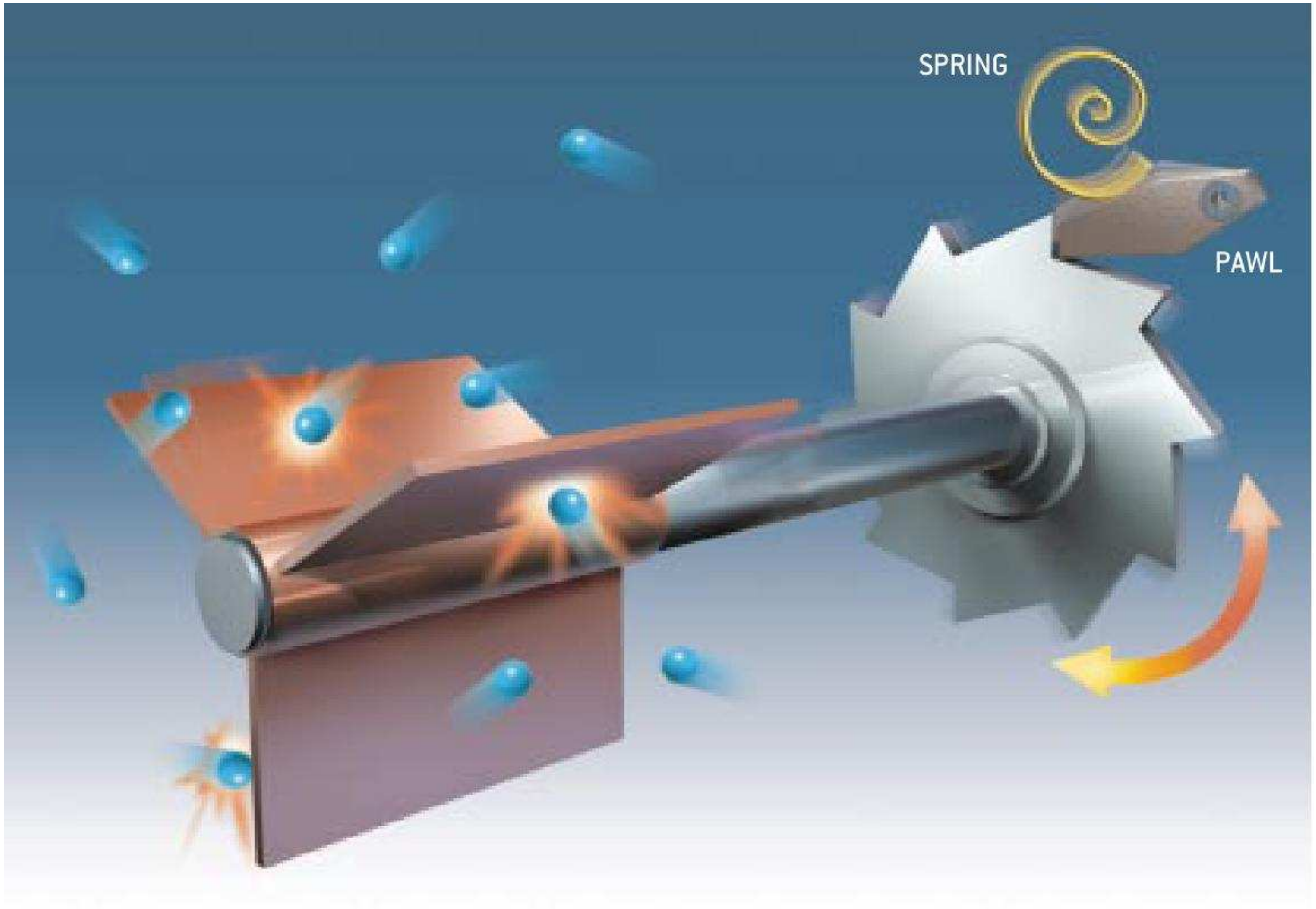
Paradoxical Noise-Effects far from Thermal Equilibrium

Peter Reimann
Universität Bielefeld

- **Ratchet Effects**
- **Negative Mobility in a Microfluidic Device**
- **Sorting Chiral Particles**

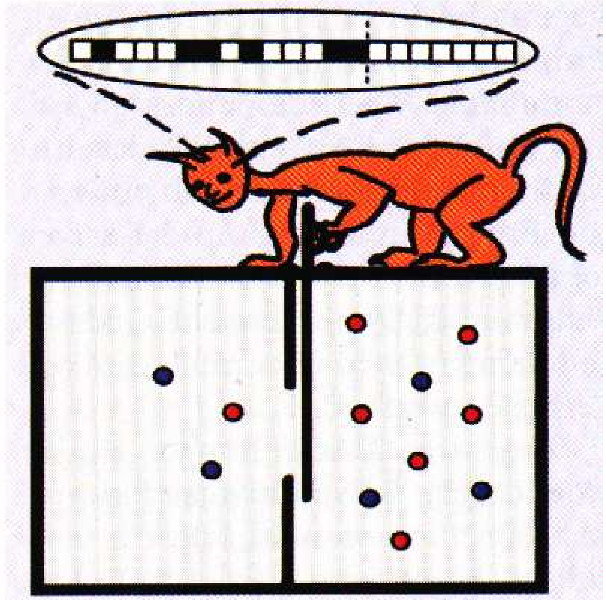
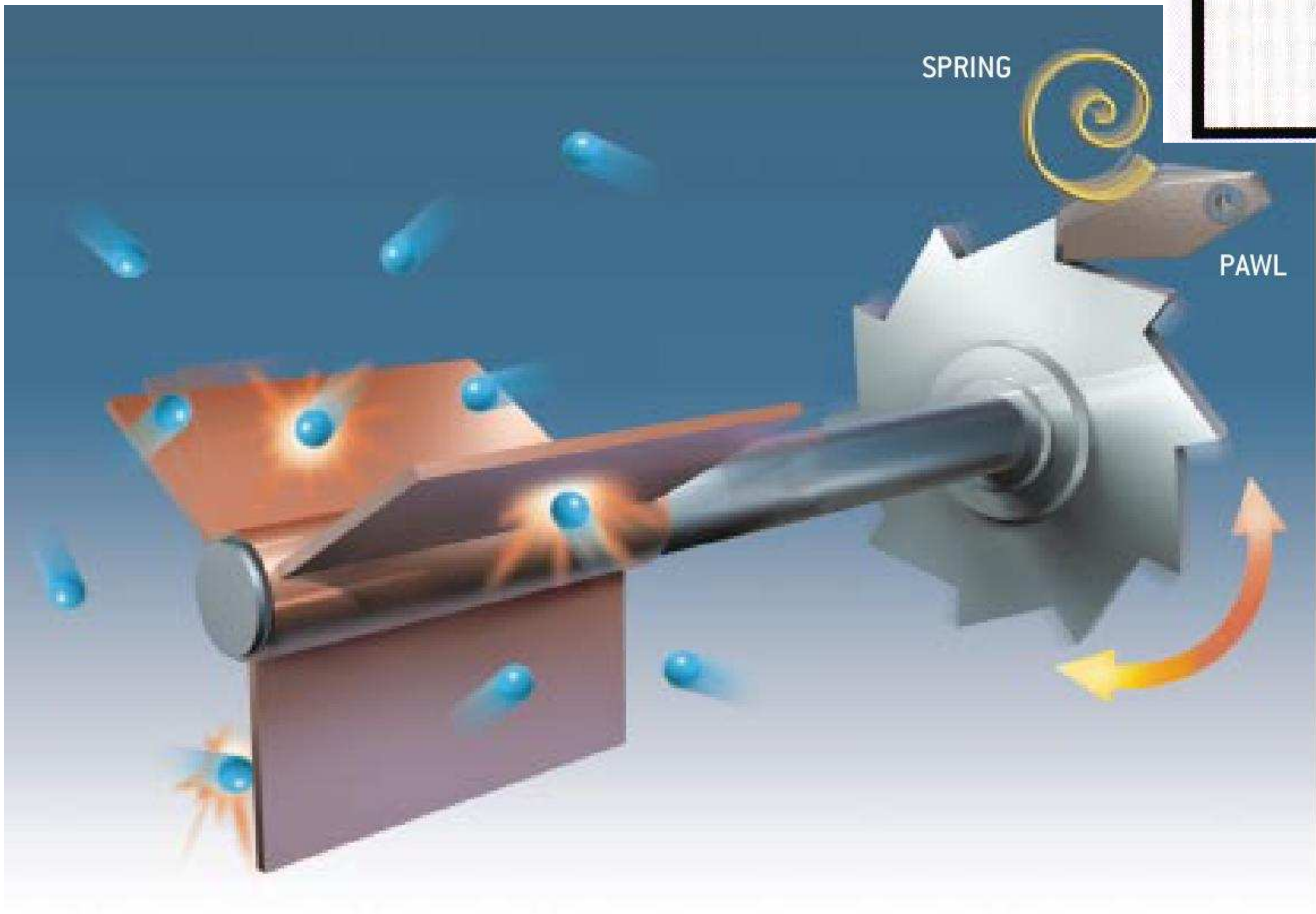
Ratchet and Pawl

[Smoluchowski 1912, Feynman 1963]



Ratchet and Pawl

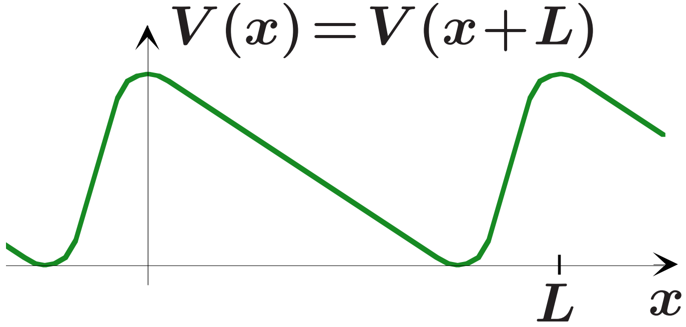
[Smoluchowski 1912, Feynman 1963]

Model

$$m\,\ddot{x} = -\gamma\,\dot{x} - V'(x) + \xi(t)$$

$m \to 0$: $$\boxed{\gamma\,\dot{x} = -V'(x) + \xi(t)}$$



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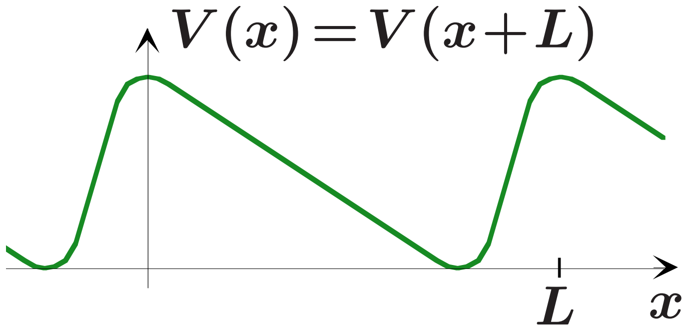

It follows:

$\boldsymbol{\xi(t)}$ Gauss , $\boldsymbol{\langle \xi(t) \rangle = 0}$, $\boldsymbol{\langle \xi(t)\,\xi(s) \rangle = 2\gamma kT\,\delta(t-s)}$

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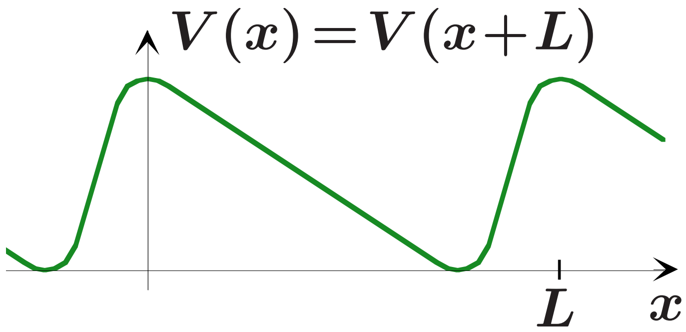

It follows:

$$\boxed{\xi(t) \text{ Gauss }, \quad \langle \xi(t) \rangle = 0 \;, \quad \langle \xi(t)\,\xi(s) \rangle = 2\gamma k T\,\delta(t-s)}$$

Theory of Fokker-Planck processes:

$$\boxed{\langle \dot{x} \rangle := \lim_{t\to\infty} \frac{x(t)}{t} = 0} \quad \text{(2nd law)}$$

Model

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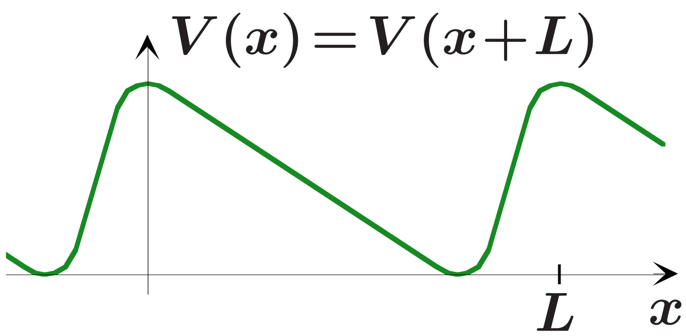

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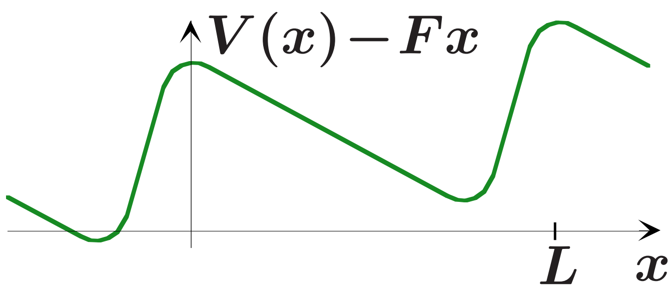
Theory of Fokker-Planck processes:

$$\boxed{\langle \dot{x} \rangle := \lim_{t\to\infty} \frac{x(t)}{t} = 0}$$ (2nd law)

Generalization:

$$\gamma\,\dot{x} = -V'(x) + F + \xi(t)$$

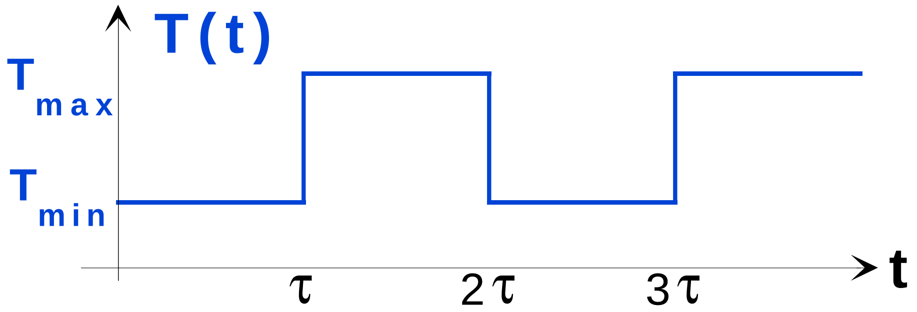
$$\boxed{F < 0 \ \Rightarrow\ \langle \dot{x} \rangle < 0 \quad \text{for any} \quad T > 0}$$



Temperature Ratchet

$$\gamma\,\dot{x} = -V'(x) + \xi(t) \; + \; F$$

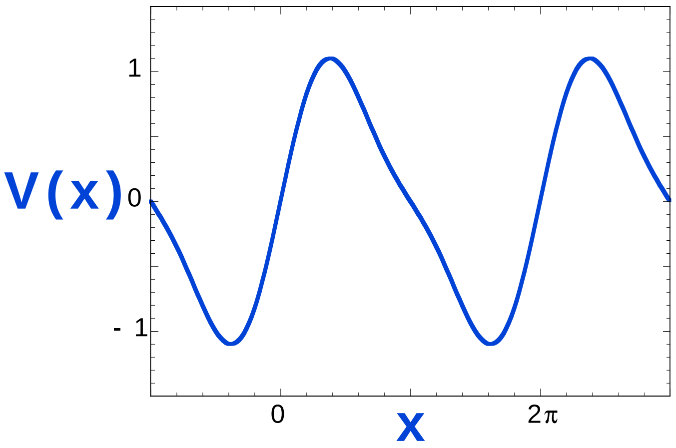
$$\langle \xi(t)\,\xi(s) \rangle = 2\gamma k \boldsymbol{T(t)}\,\delta(t-s)$$



dimensionless units:

$kT_{max} = 3$, $kT_{min} = 0.5$,

$\tau = 5$, $\gamma = 1$



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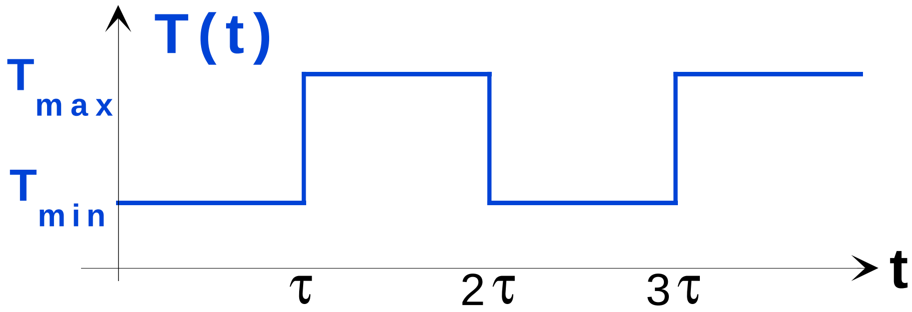

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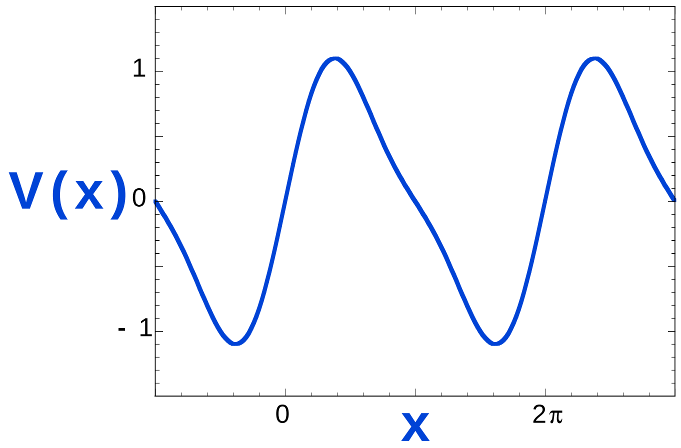
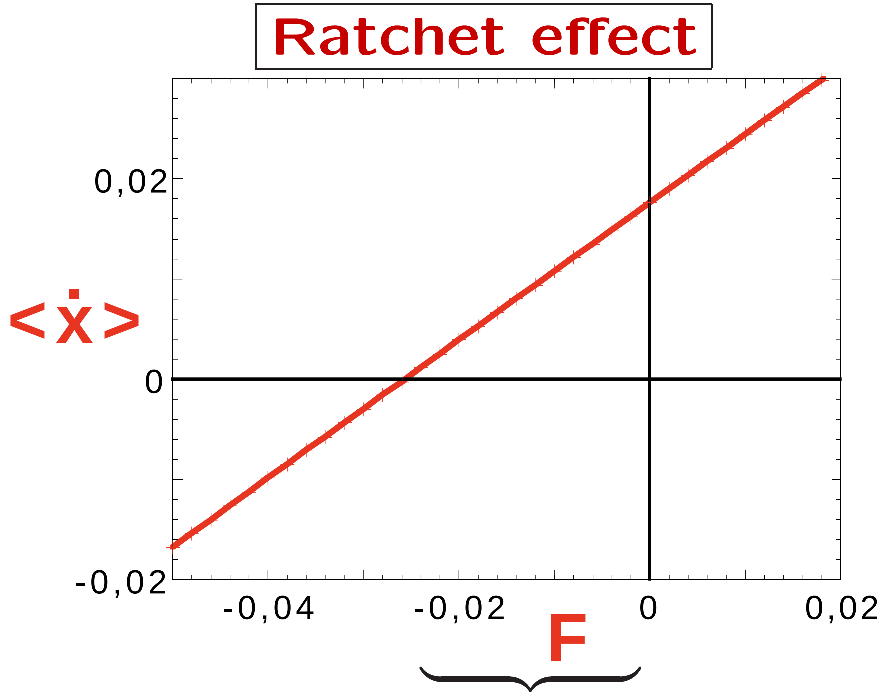
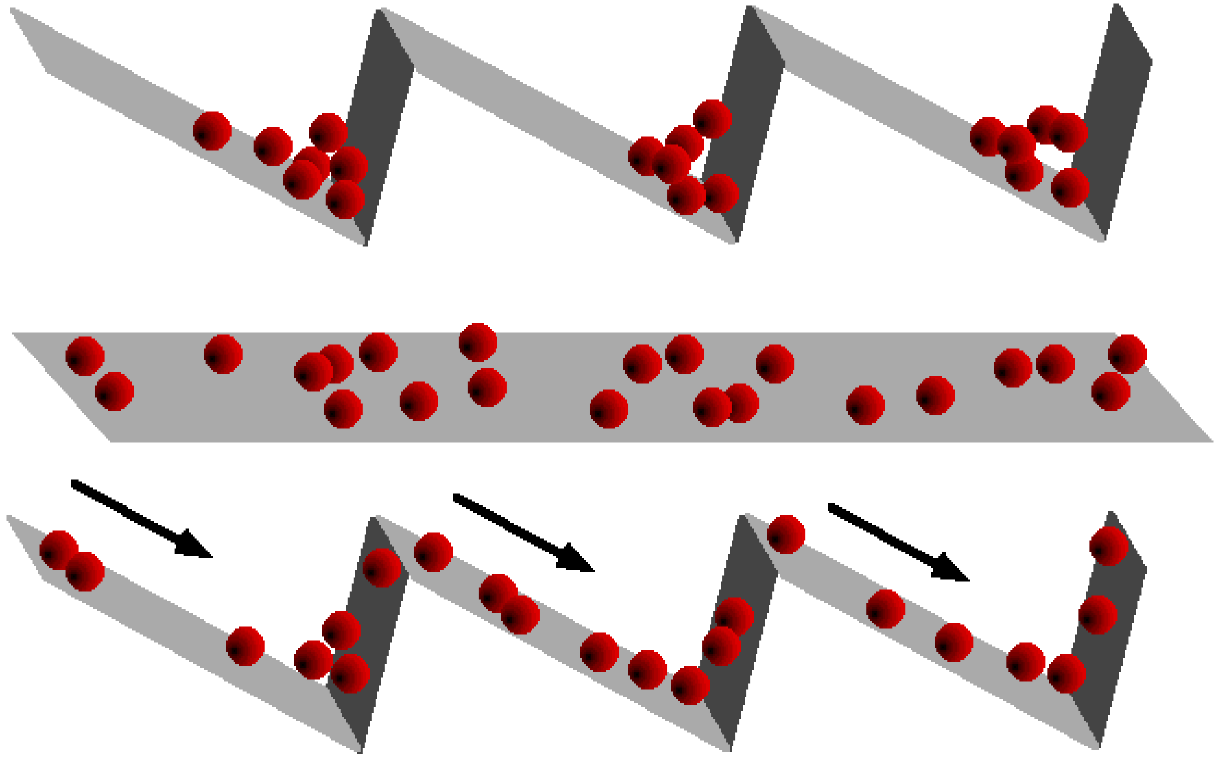




$F < 0,\ \langle \dot{x} \rangle > 0$

Explanation

($F = 0$, $kT_{\text{min}} \ll \Delta V$, $kT_{\text{max}} \gg \Delta V$, τ large)



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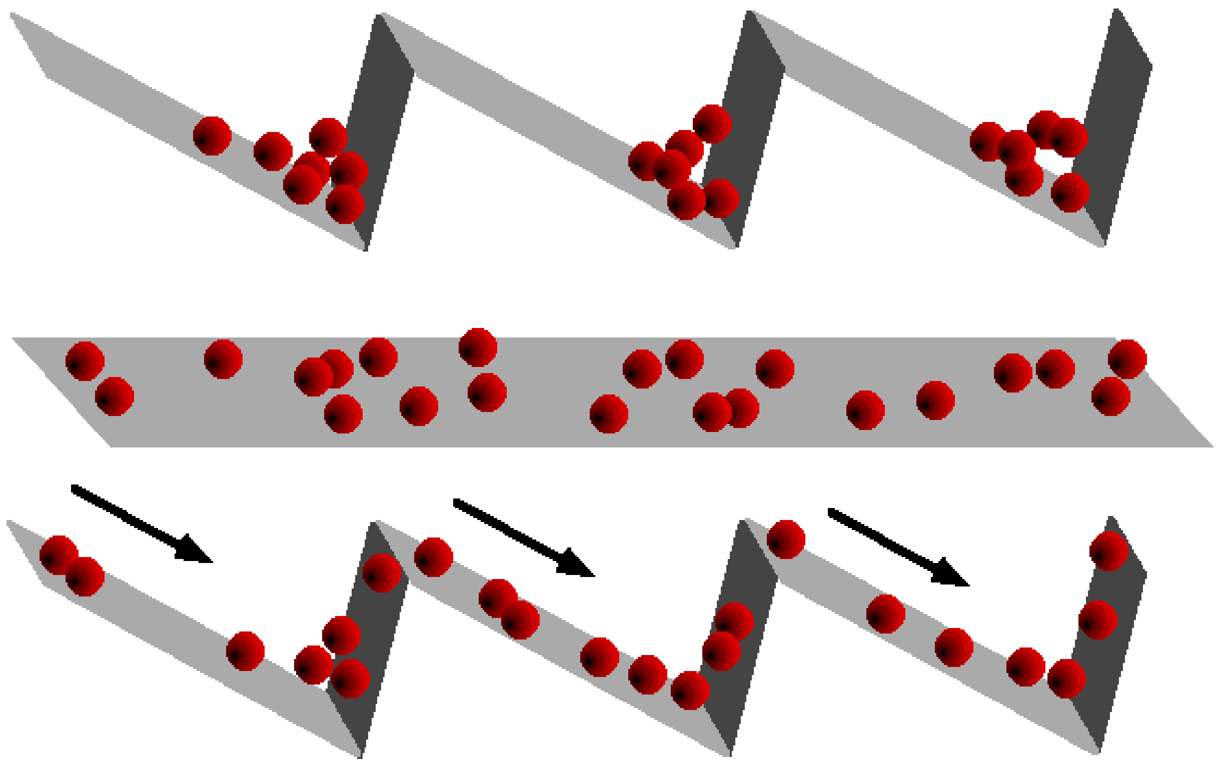

- **Particle pump**
- **Mechanism robust (provided τ large)**
- **No contradiction to 2nd law**
- **$x(t)$ and $T(t)$ "loosely coupled"**

Transport Direction

$$\gamma\,\dot{x} = -V'(x) + \xi(t) \quad , \quad \langle \xi(t)\,\xi(s)\rangle = 2\gamma k T(t)\,\delta(t-s)$$

Consider $\langle \dot{x} \rangle$ as a function of an arbitrary parameter μ $(\gamma, \tau, T_{min}, ...)$ and choose μ_0 arbitrarily.

There exists a $V(x)$ with a current inversion at μ_0

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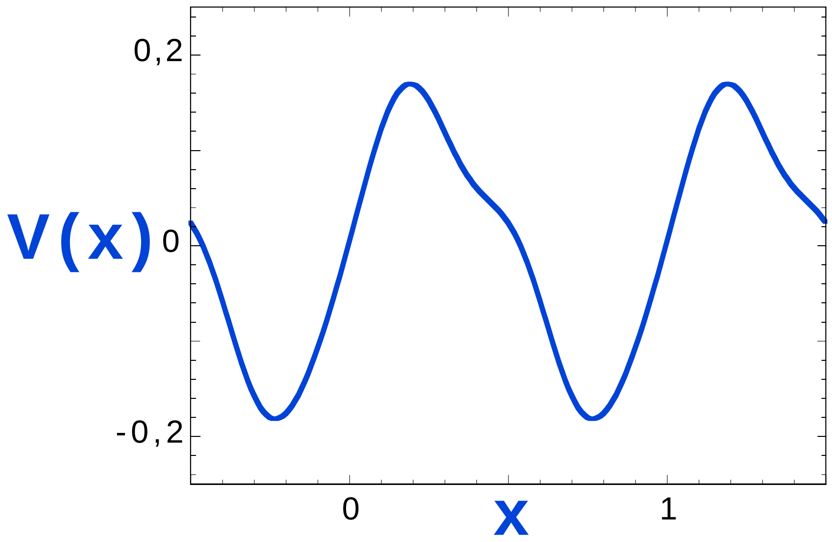
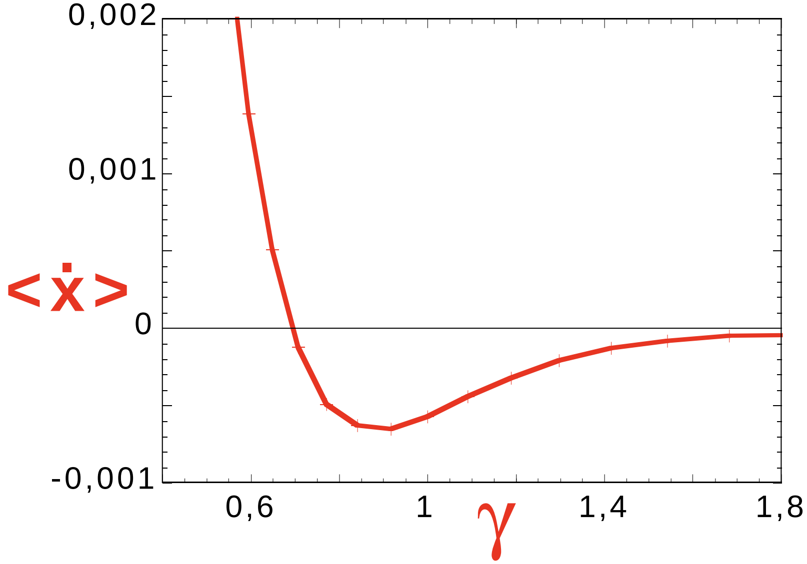
Example: $\mu = \gamma\,,\ \mu_0 = 0.7\,,\ kT_{max} = 0.18\,,\ kT_{min} = 0.02\,,\ \tau = 0.02$





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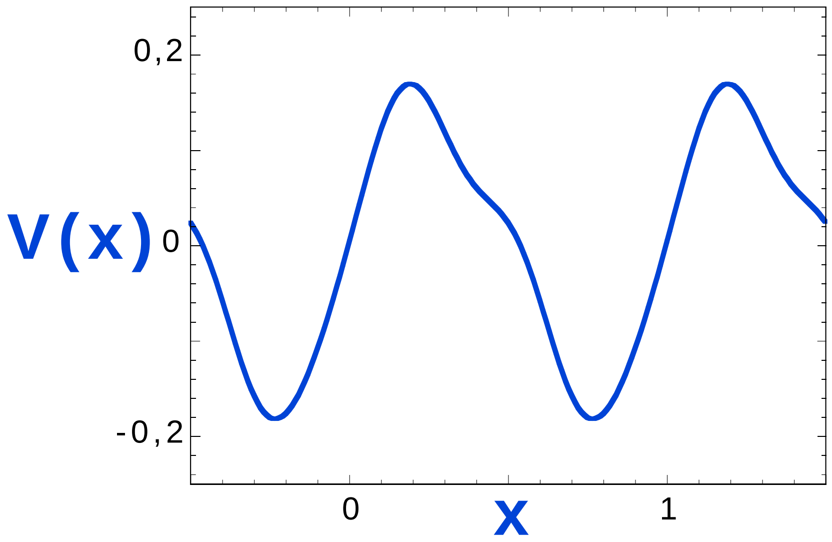
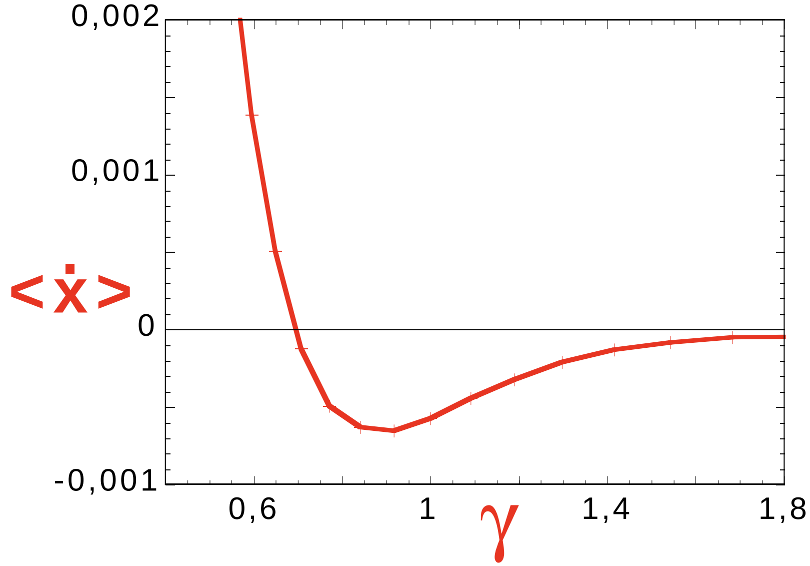
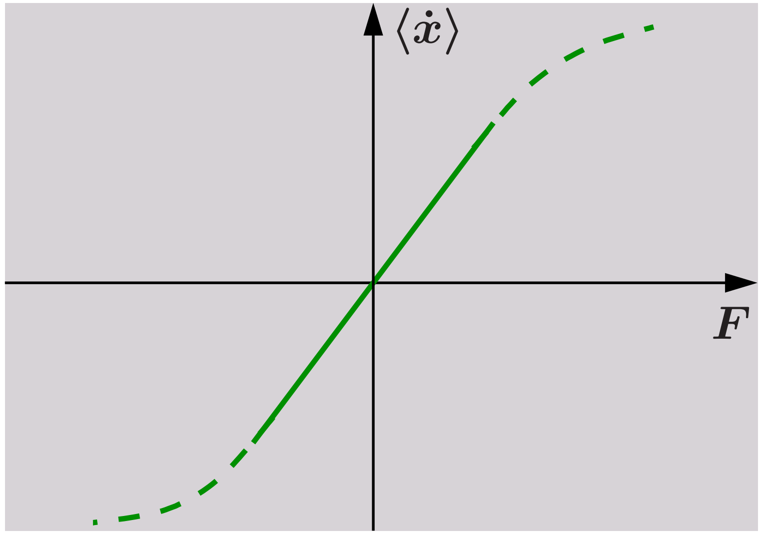




e. g. $\gamma = 6\pi\eta r$ $\Rightarrow$ **particle separation**

Negative Mobility

equilibrium response



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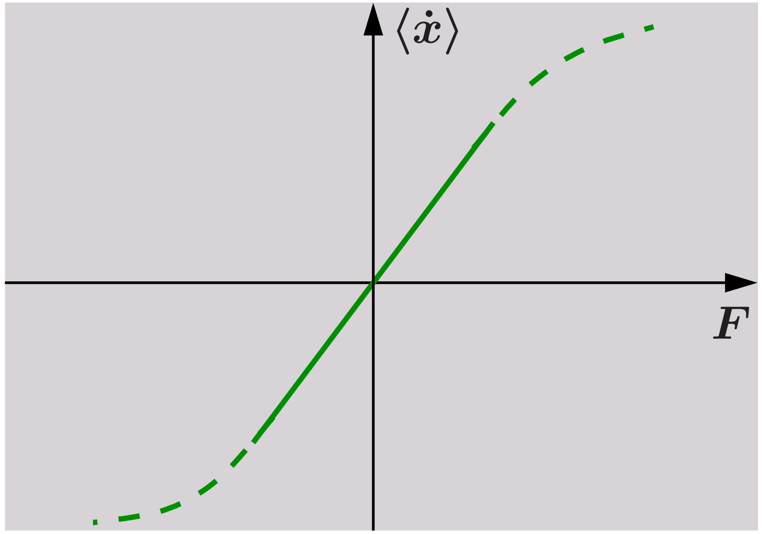
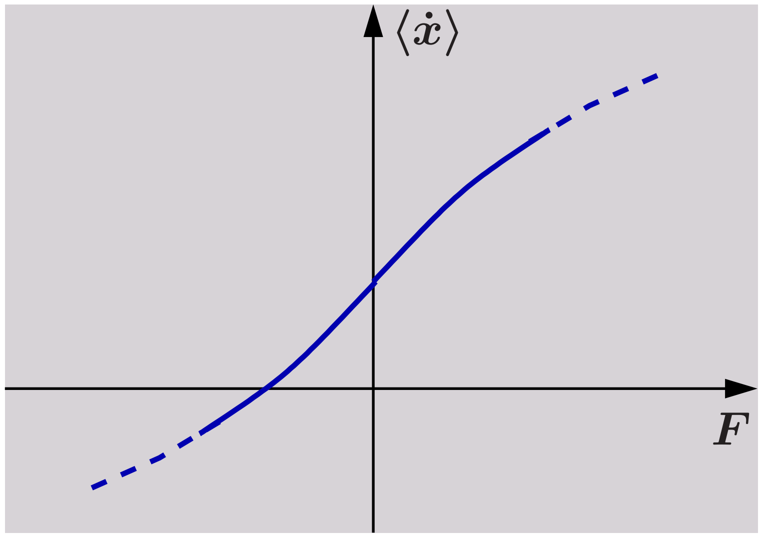


ratchet effect



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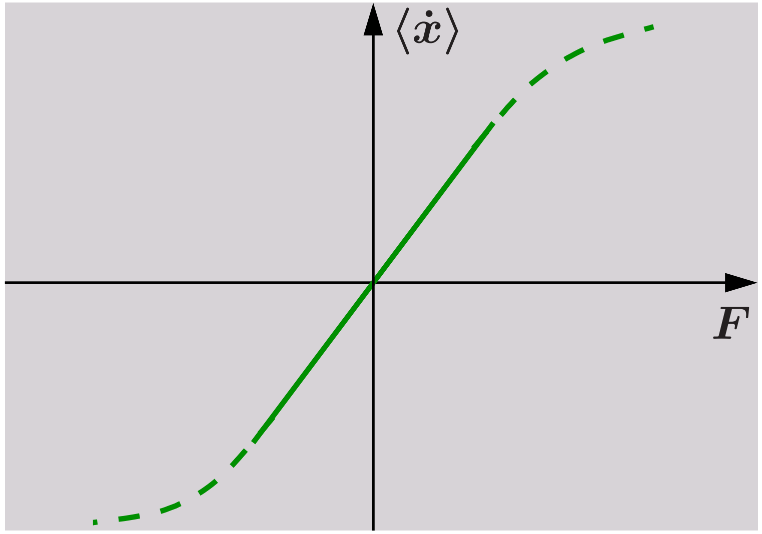

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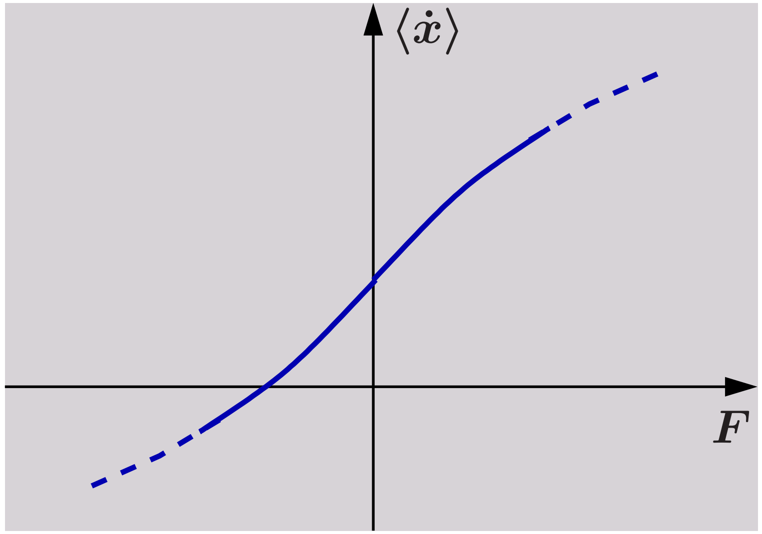
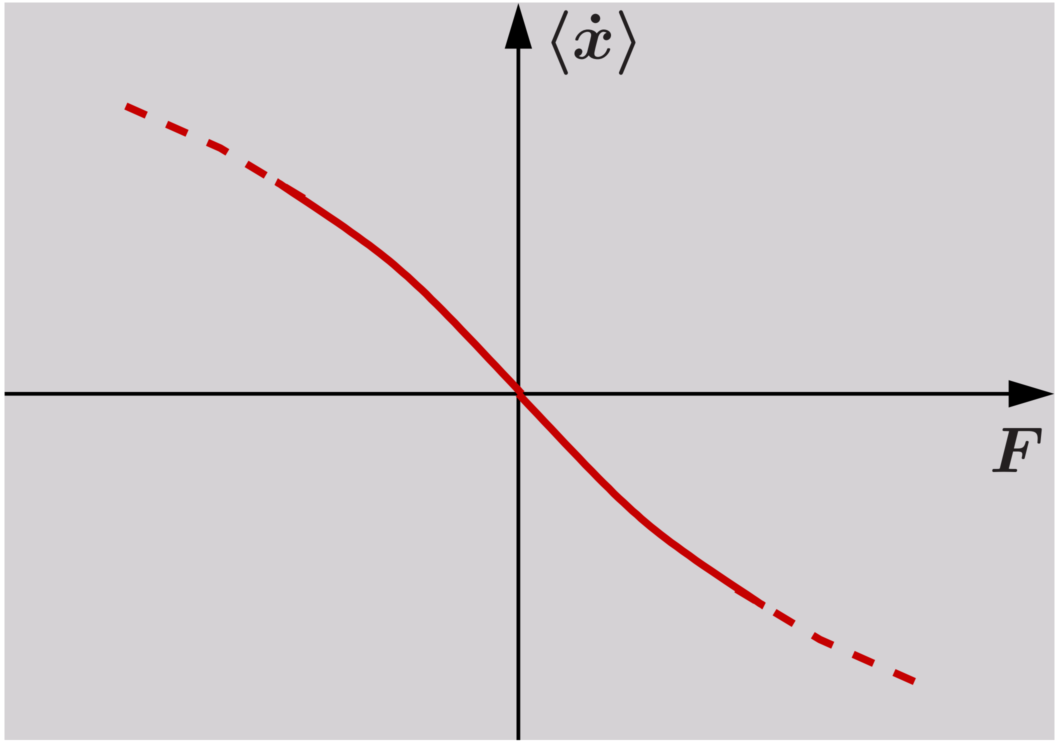


negative mobility



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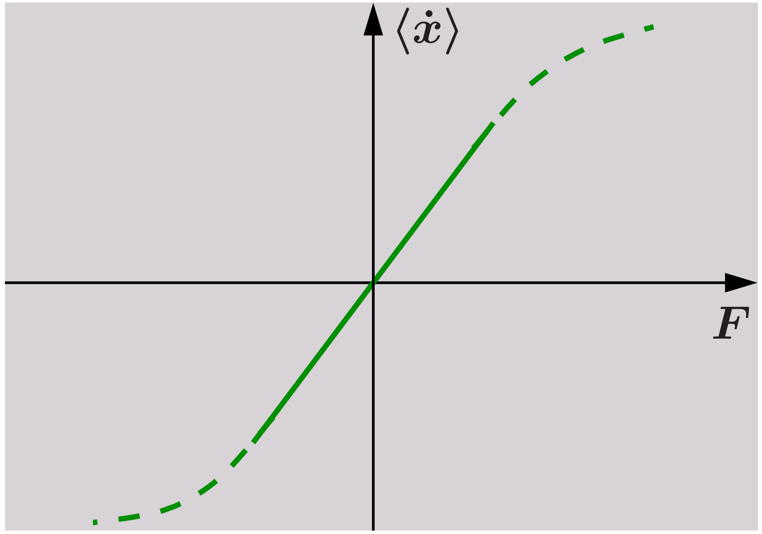

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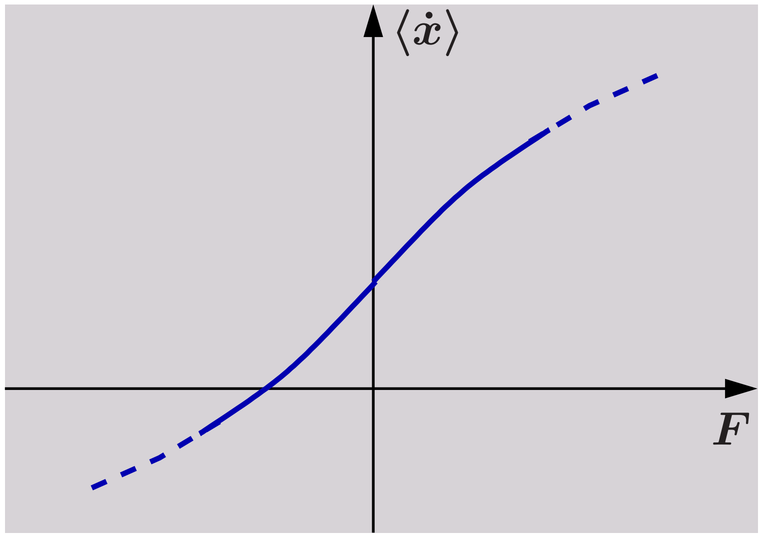
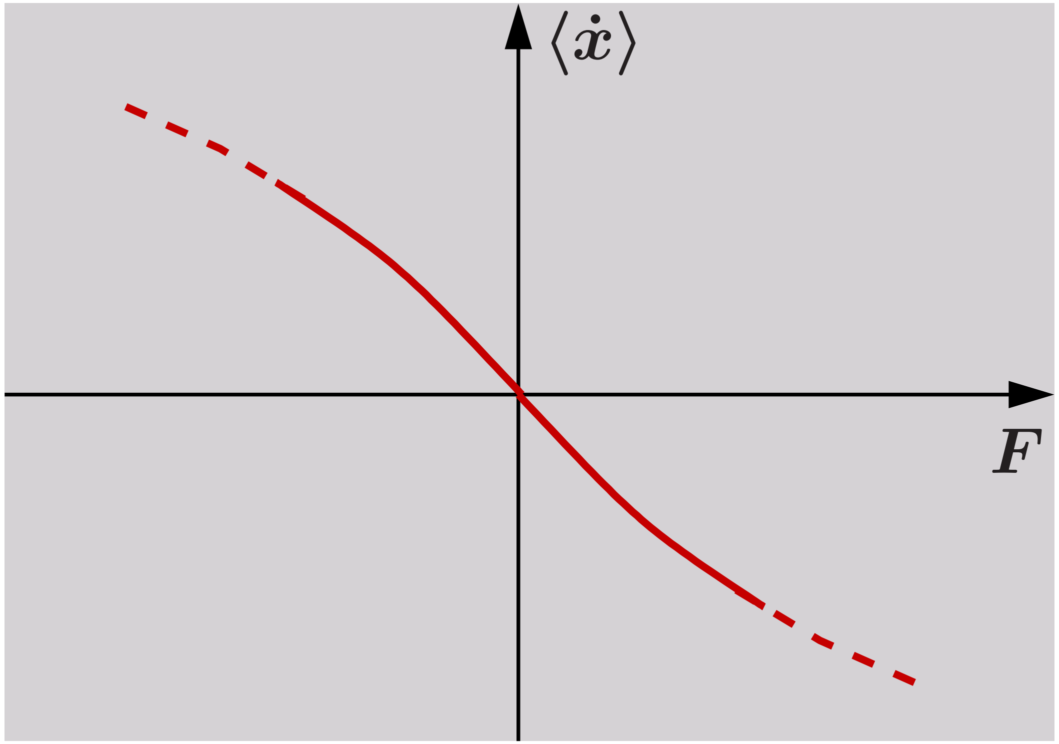


negative mobility

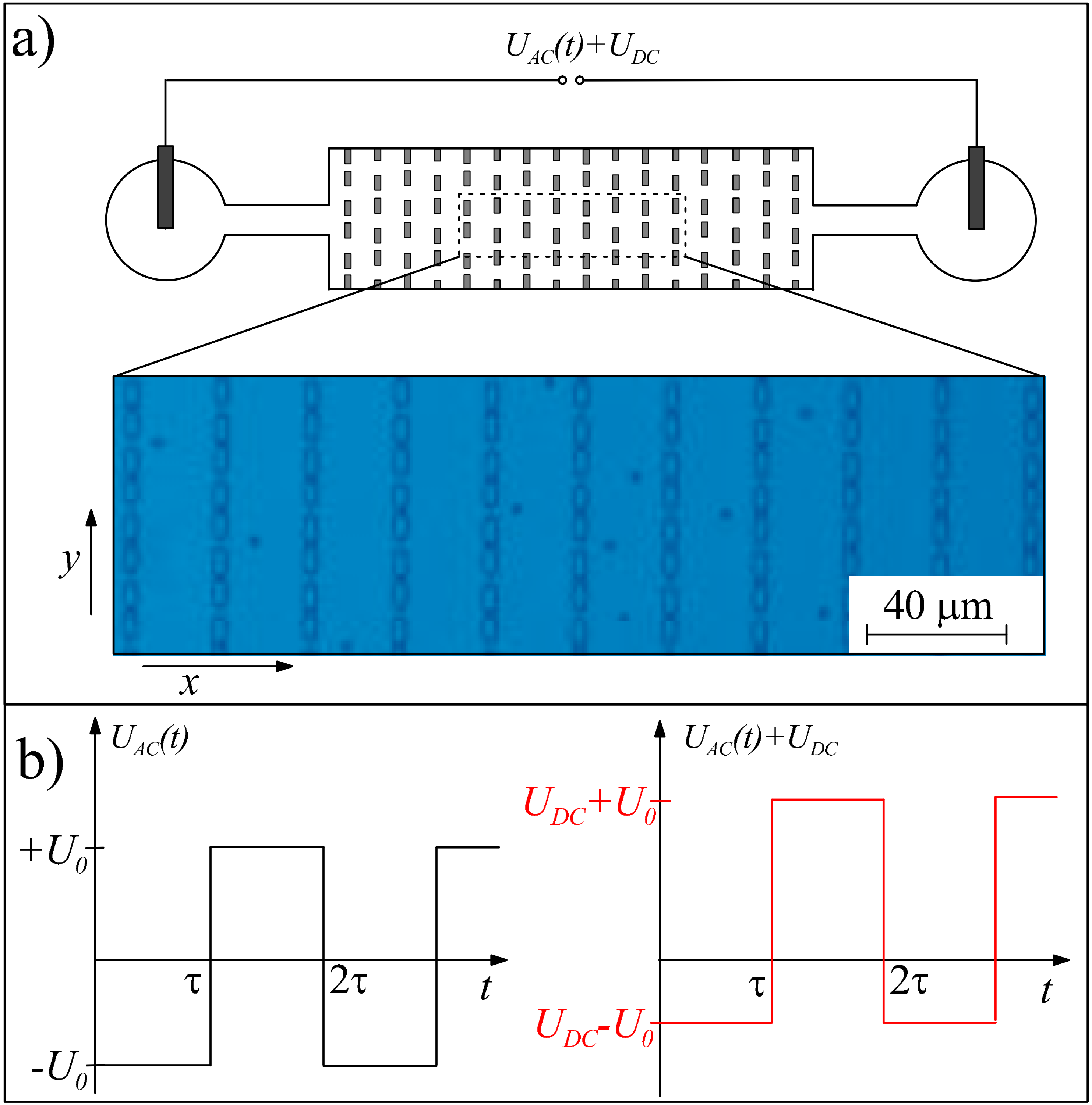


as a quantum effect:

Keay et al., PRL **75**, 4102 (1995)

as a classical effect:

Eichhorn, Reimann, Hänggi, PRL **88**, 190601 (2002)

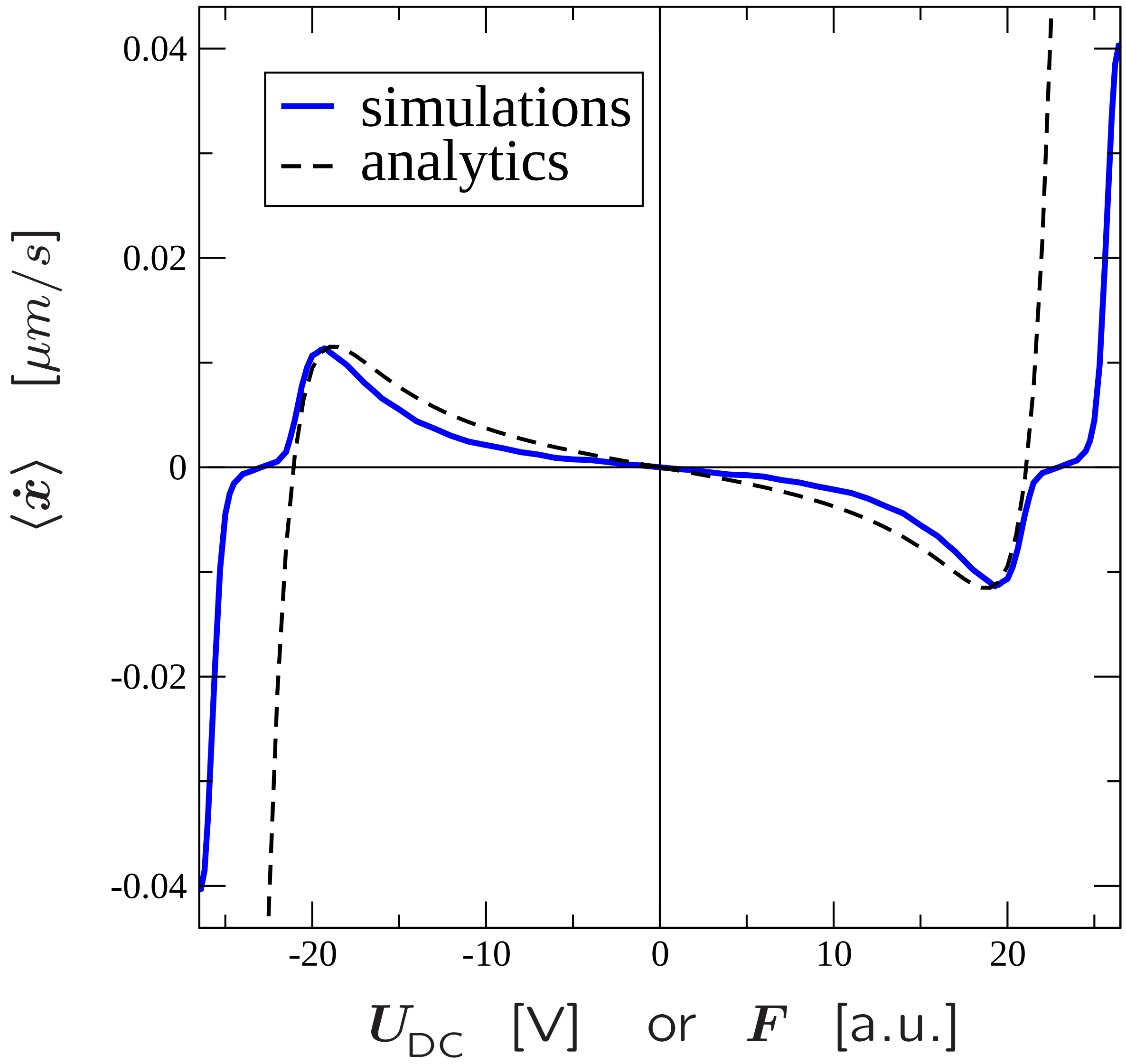


a)
$U_{AC}(t)+U_{DC}$
y
x
40 μm
b)
$U_{AC}(t)$
$+U_0$
$-U_0$
τ
2τ
t
$U_{AC}(t)+U_{DC}$
$U_{DC}+U_0$
$U_{DC}-U_0$
τ
2τ
t

Theoretical Prediction

[Eichhorn & Reimann, 2005]

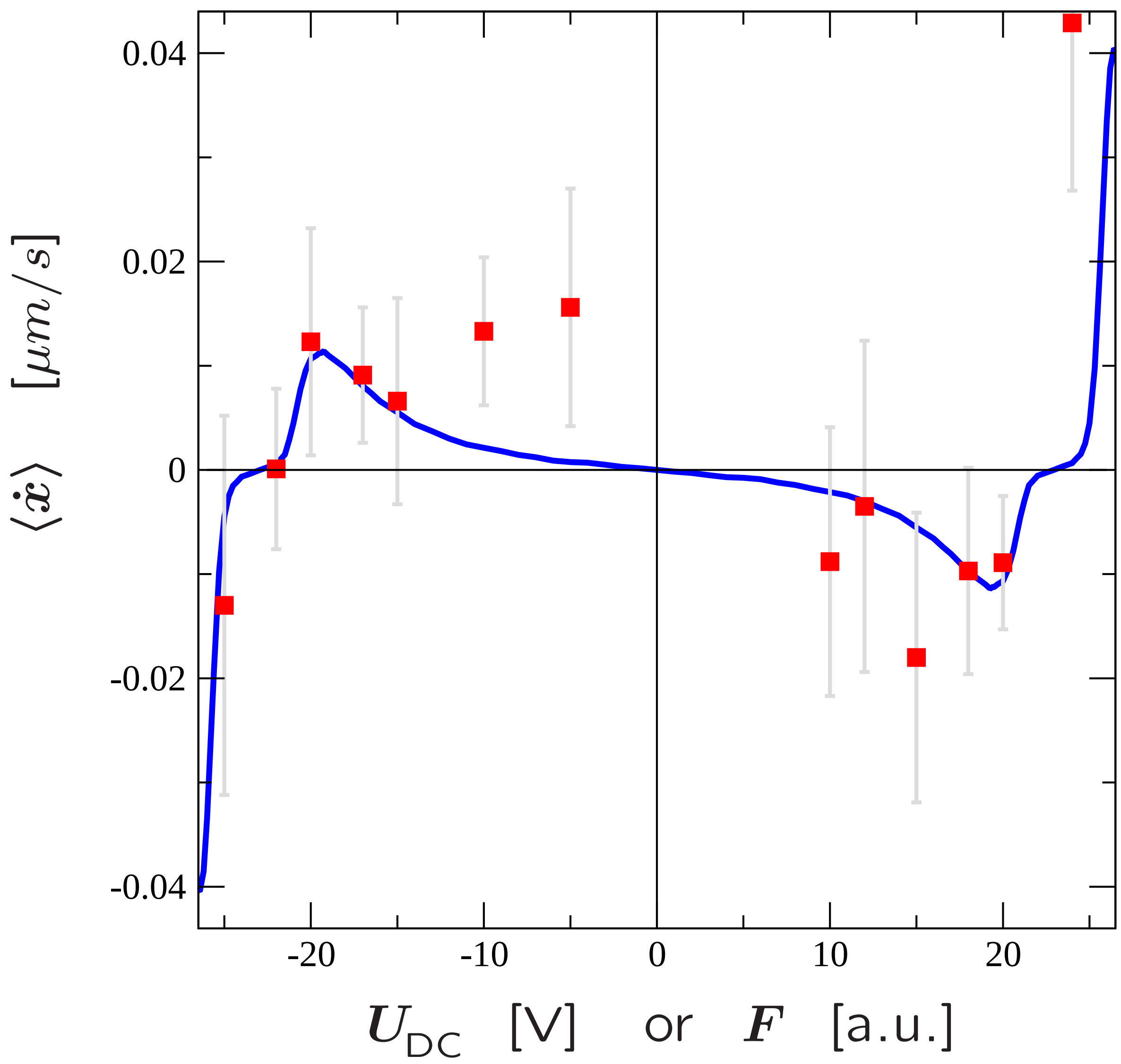
(2 μm particle diameter, $U_0 = 30$ V, $\tau = 25$ s)



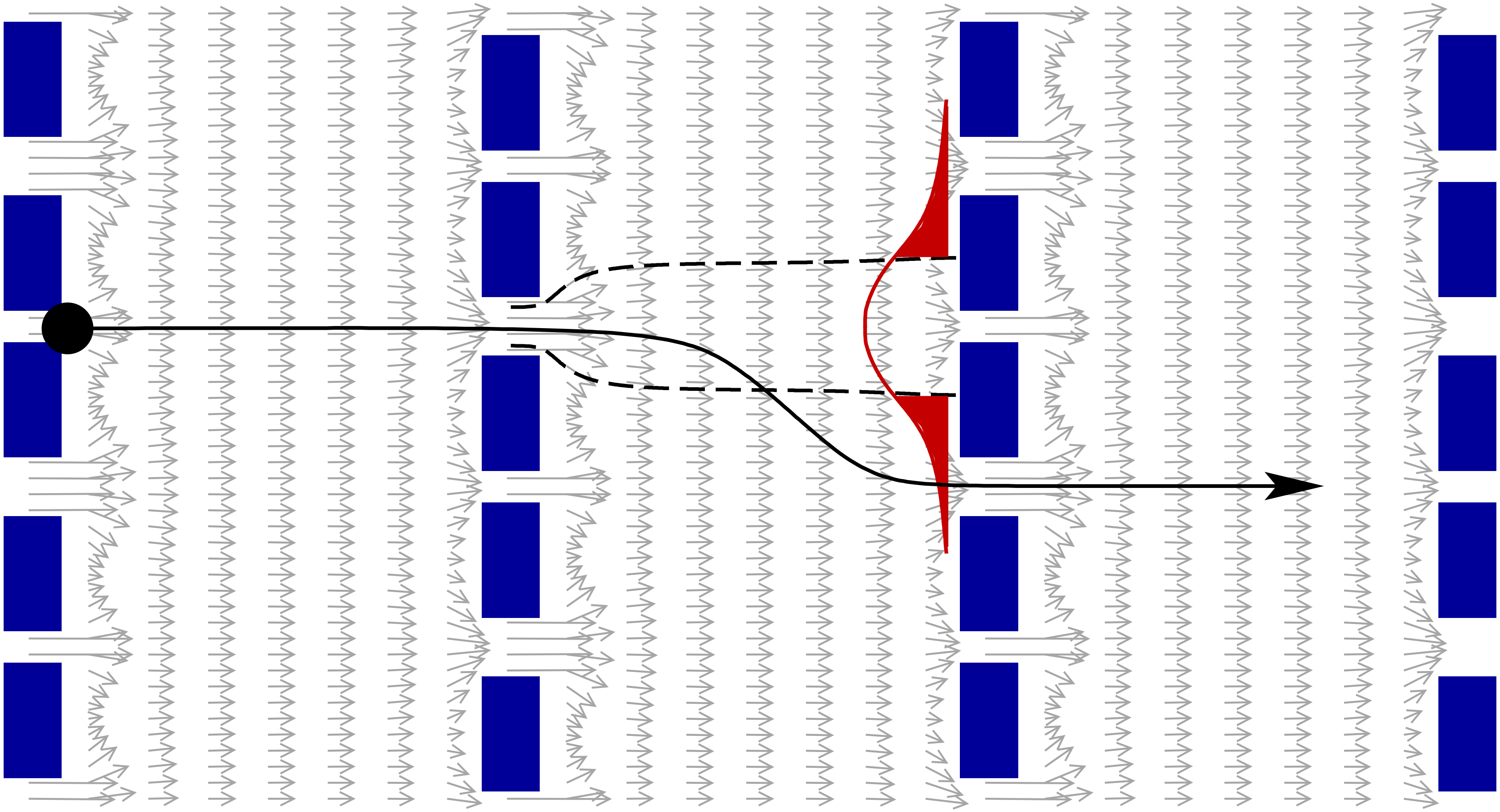
Experiment versus Theory

Experiment: Regtmeier, Ros, Anselmetti **Theory**: Eichhorn, Reimann

[Nature 436, 928 (2005)]

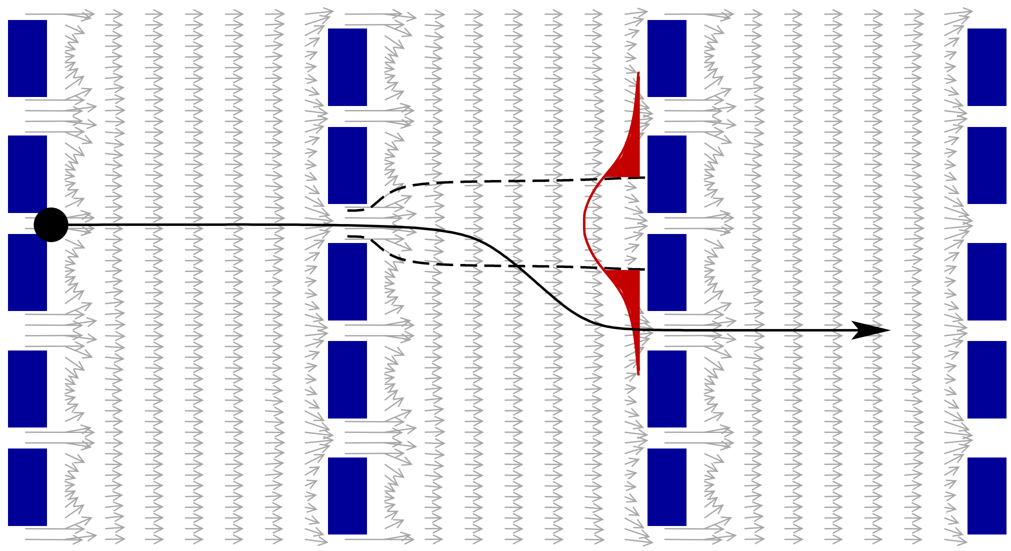


Physical Mechanism

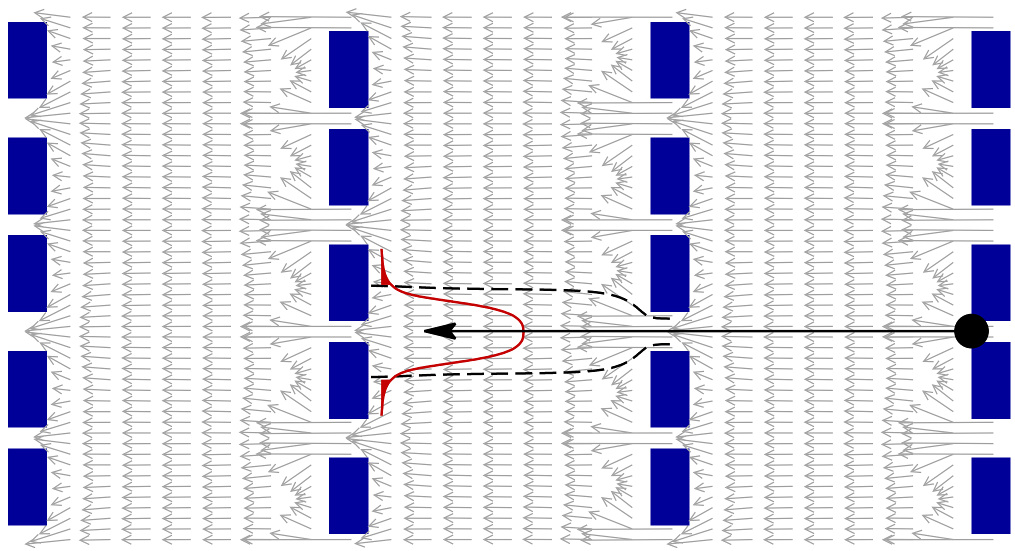


$$U_{DC} < 0\,, \quad U_{DC} + U_0 > 0$$

Physical Mechanism



$$U_{DC} < 0\,, \quad U_{DC} + U_0 > 0$$

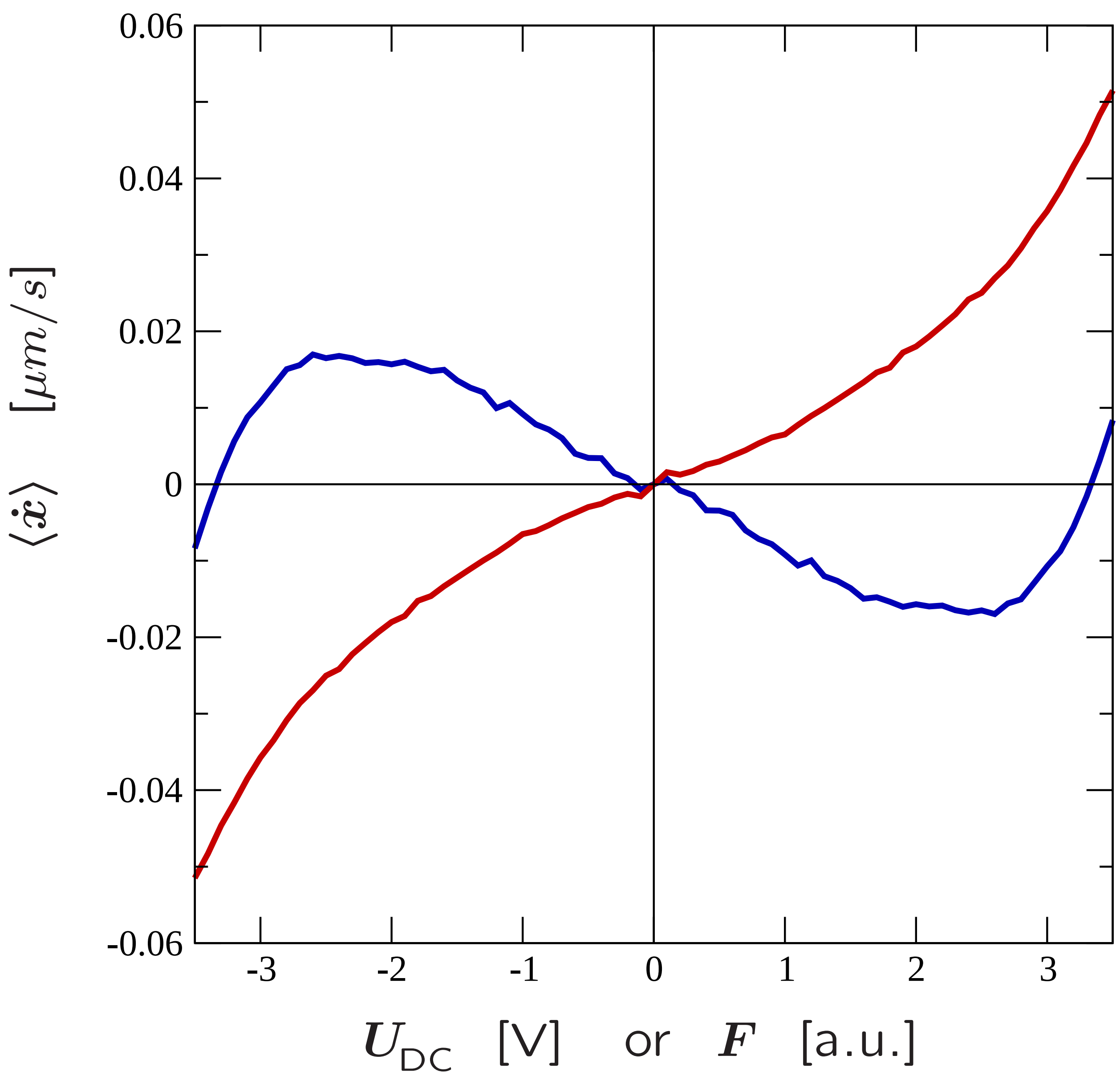


$$U_{DC} < 0\,, \quad U_{DC} - U_0 < 0$$

Theoretical Prediction

($U_0 = 6$ V, $\tau = 70$ s)

1.9 μm particles: blue **2.8 μm particles: red**



Experiment versus Theory

[Eichhorn, Regtmeier, Anselmetti, Reimann, Soft Matter **6**, 1858 (2010)]

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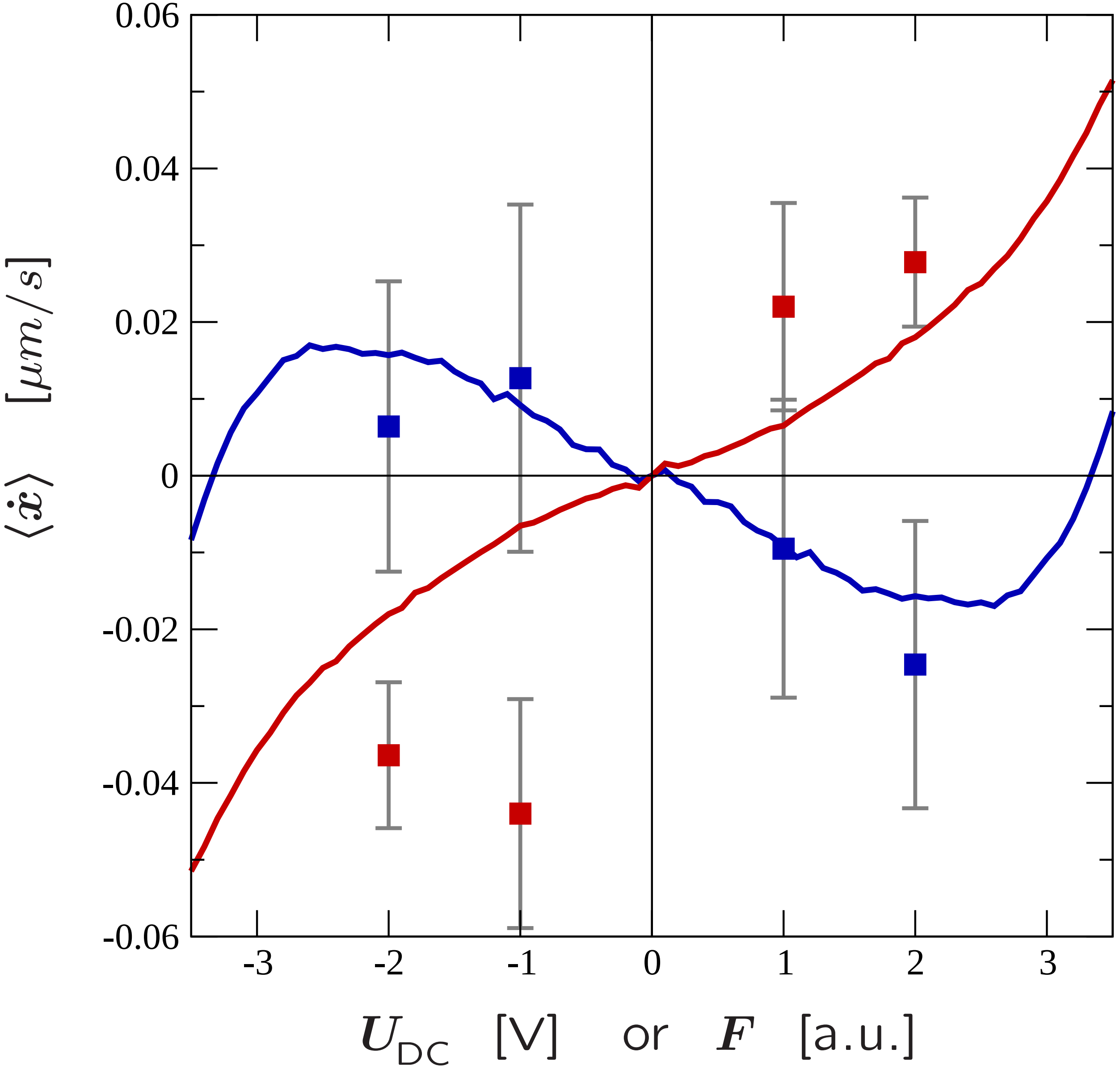
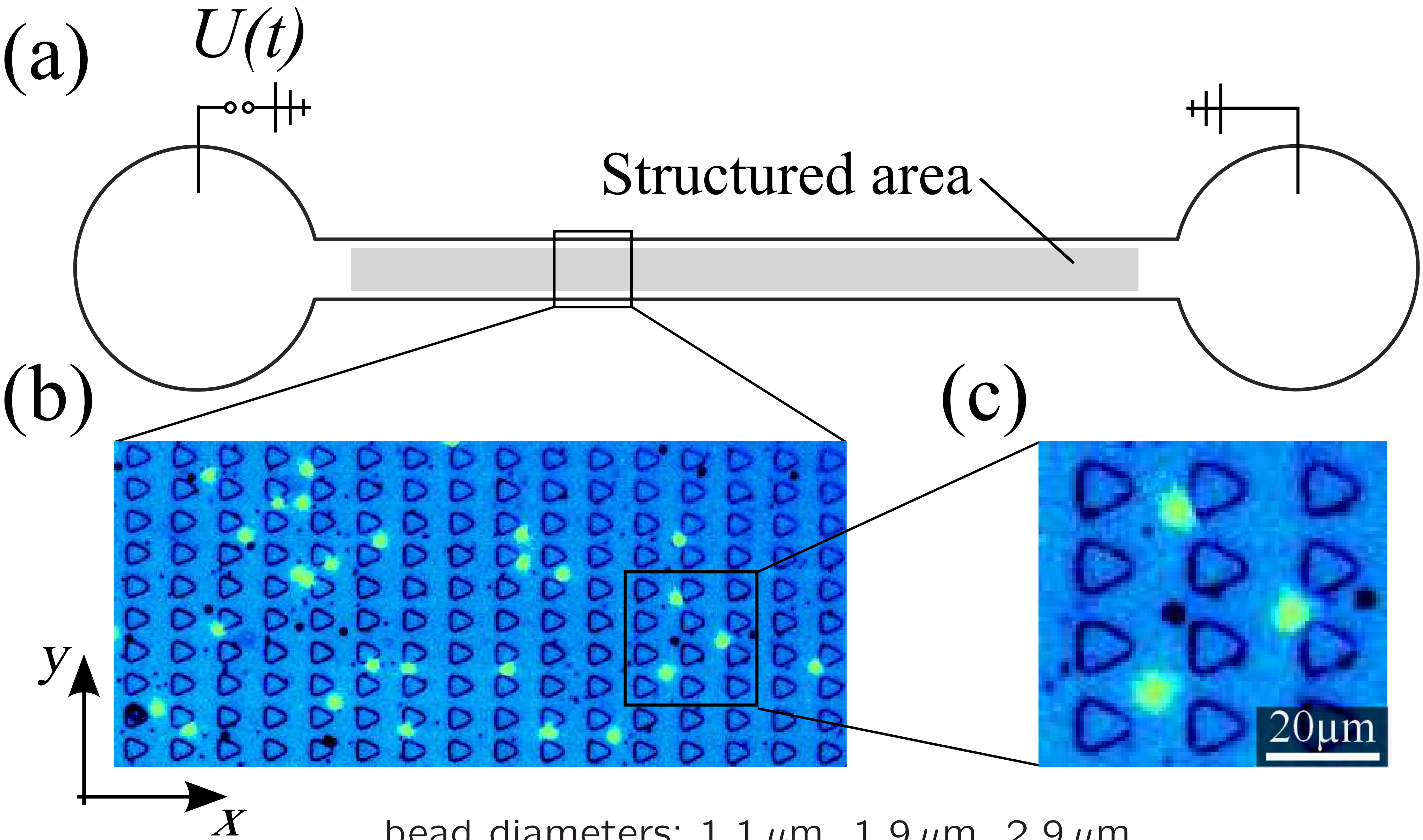


A tunable microfluidic ratchet for particle sorting

[Bogunovich, Eichhorn, Regtmeier, Anselmetti, Reimann, submitted]



bead diameters: 1.1 μm, 1.9 μm, 2.9 μm

$U(t) = U_{DC} + U_{AC} \sin(\omega t)$ $\Rightarrow$ velocity in x-direction ?

A tunable microfluidic ratchet for particle sorting

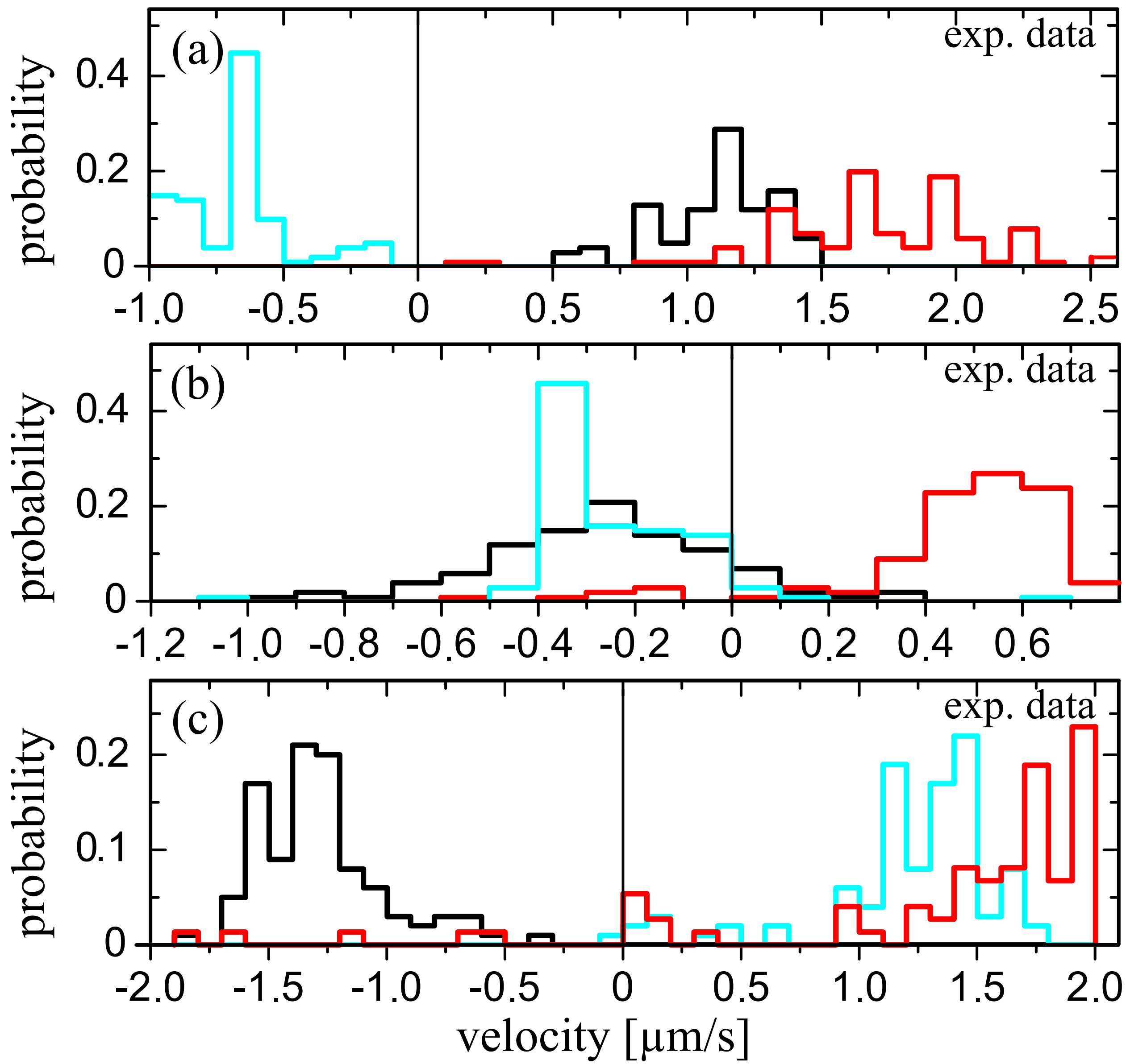
[Bogunovich, Eichhorn, Regtmeier, Anselmetti, Reimann, submitted]

$$U(t) = U_{\mathrm{DC}} + U_{\mathrm{AC}} \sin(\omega t)$$

$$\omega/2\pi = 100 \text{ Hz}$$

Δt [s]	U_{DC} [V]	U_{AC} [V]
protocol (a)		
10	0	400
50	-15	320
12	+10	0
protocol (b)		
10	0	400
90	-15	400
25	+16	330
10	-10	0
protocol (c)		
0	0	400
40	-15	360
12	+10	0

1.1 μm black, 1.9 μm red, 2.9 μm blue



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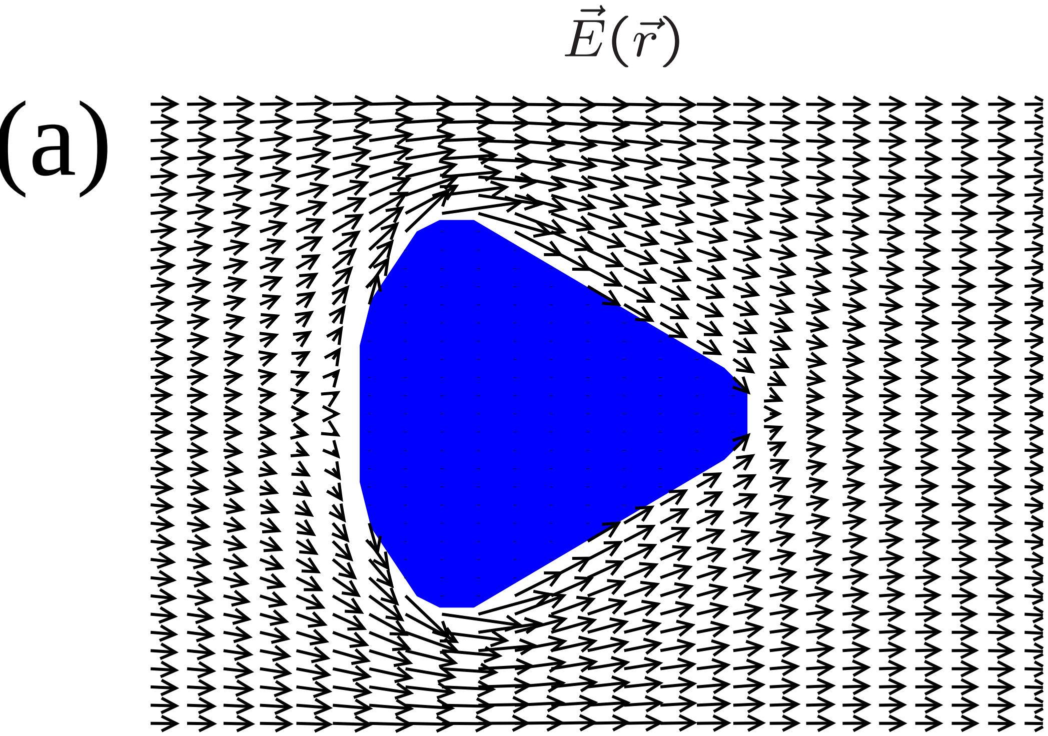
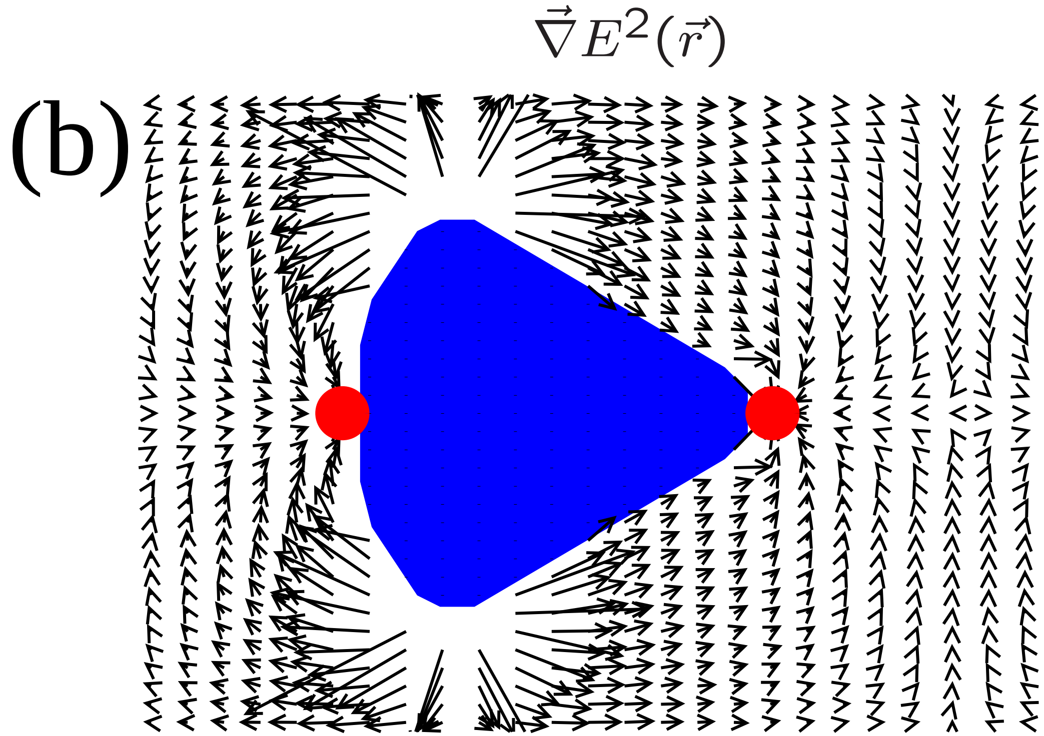




Electrophoresis: $\vec{F}(\vec{r}) = q\,\vec{E}(\vec{r})$

Dielectrophoresis: $F(\vec{r}) = \vec{\nabla}[\vec{p}\cdot\vec{E}(\vec{r})]$

$\vec{p} = \alpha\vec{E}(\vec{r})$

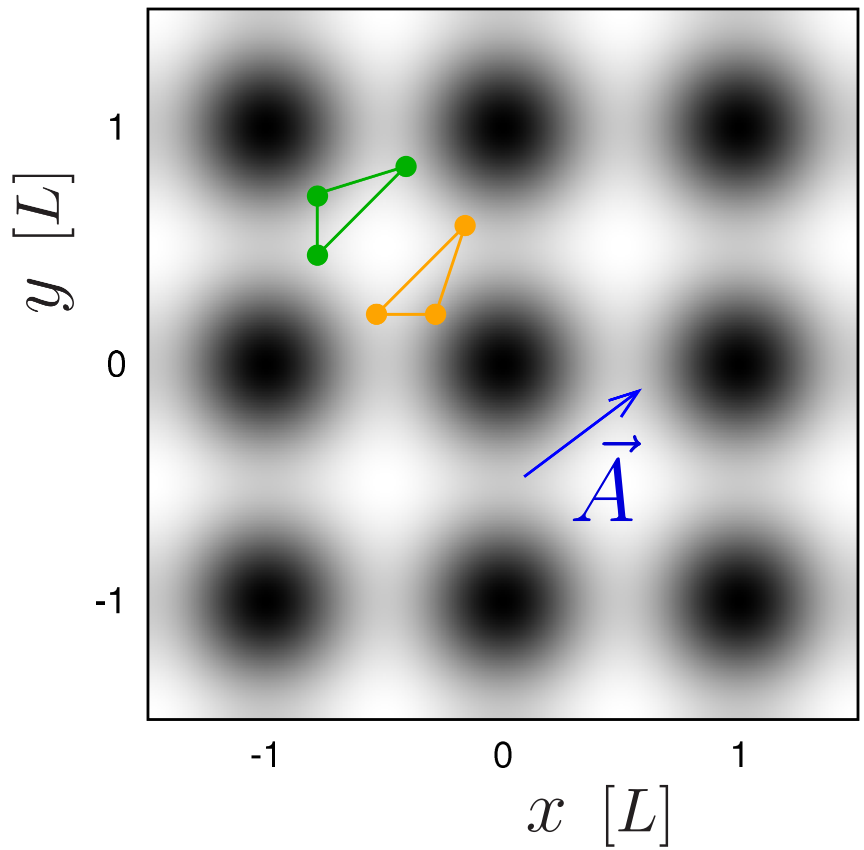
$U(t) = U_{\mathrm{DC}} + U_{\mathrm{AC}}\,\sin(\omega t) \quad \Rightarrow \quad$ time averaged forces:

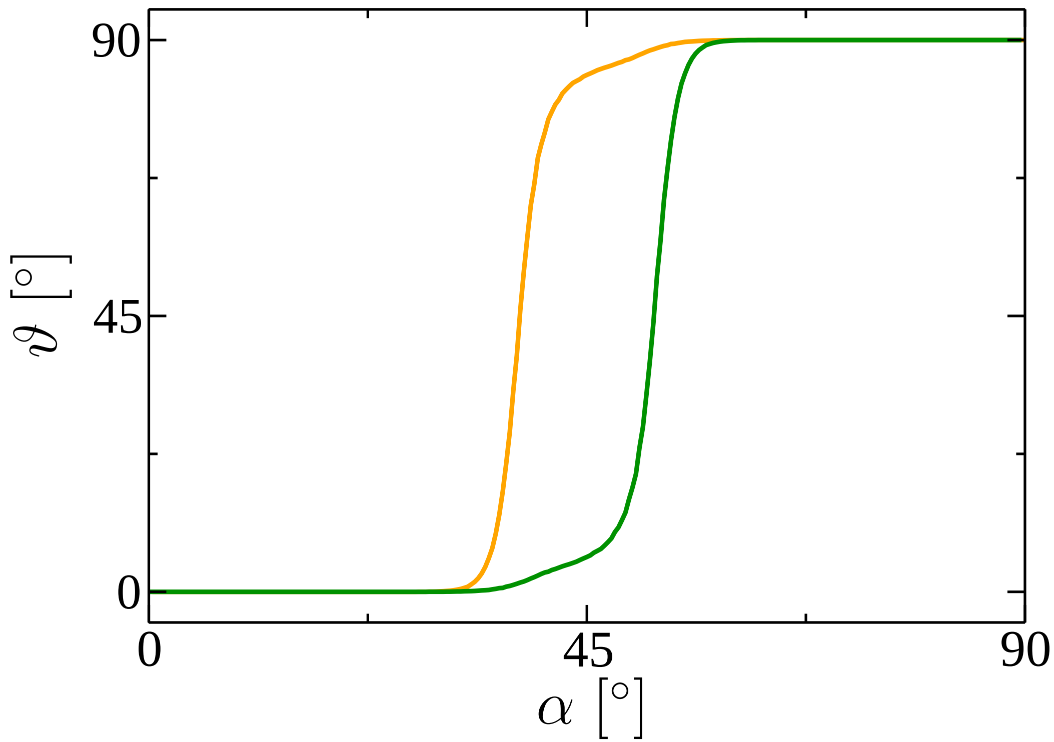
$\propto\ q\,U_{\mathrm{DC}}$

$\propto\ \alpha\,[U_{\mathrm{DC}}^2 + U_{\mathrm{AC}}^2/2]$

Sorting Chiral Particles

[Speer, Eichhorn, Reimann, PRL **105**, 090602 (2010)]





2d square lattice, $L = 1$

Gaussian "potential hills"

$\sigma = L/4$, unit potential barriers

static force $\vec{A} = A\,\vec{e}_\alpha$

$\vec{e}_\alpha := \vec{e}_x \cos(\alpha) + \vec{e}_y \sin(\alpha)$

rigid triangles (chiral)

overdamped dynamics

$\gamma = 1,\ kT = 0.014$

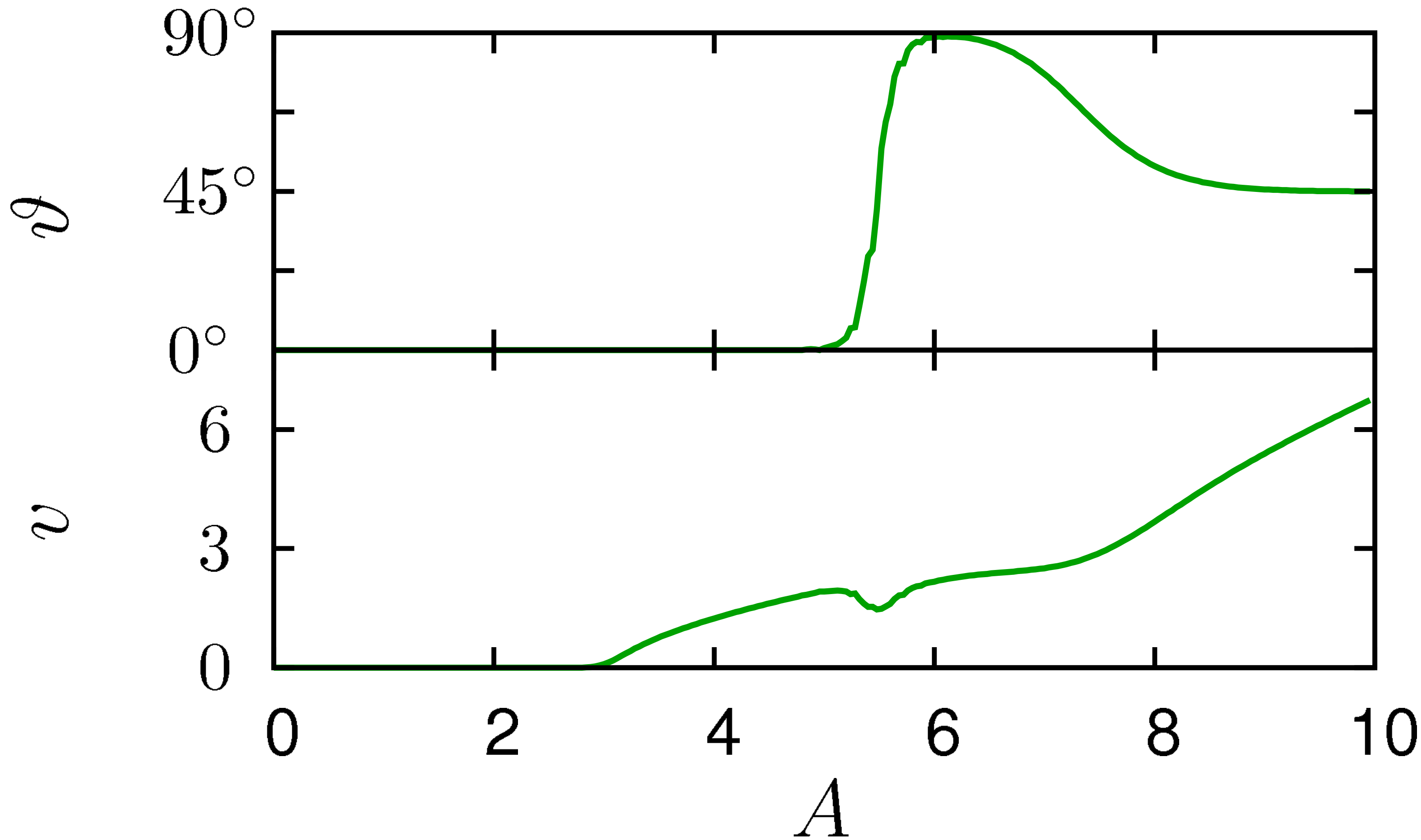
average velocity $\vec{v} = v\,\vec{e}_\vartheta$

Sorting Chiral Particles

[Speer, Eichhorn, Reimann, PRL **105**, 090602 (2010)]

static force $\vec{A} = A\,\vec{e}_\alpha$ with $\boxed{\alpha = 45^\circ}$

resulting average velocity $\vec{v} = v\,\vec{e}_\vartheta$:

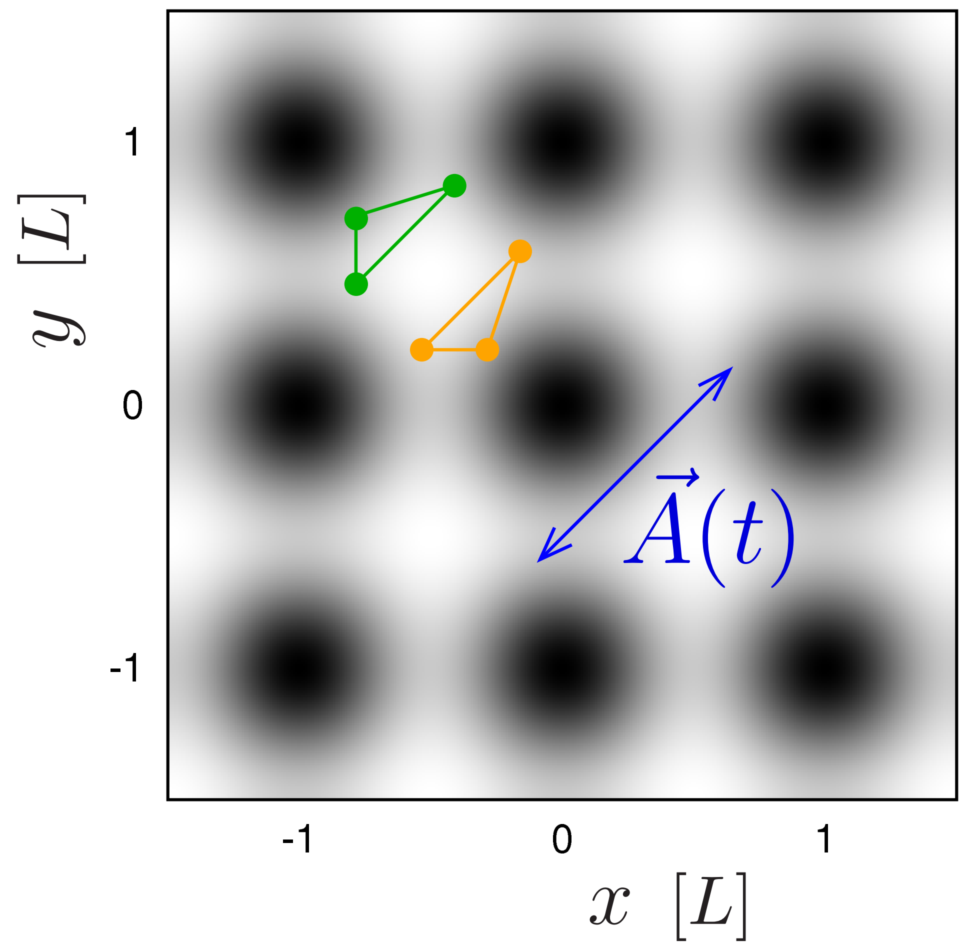


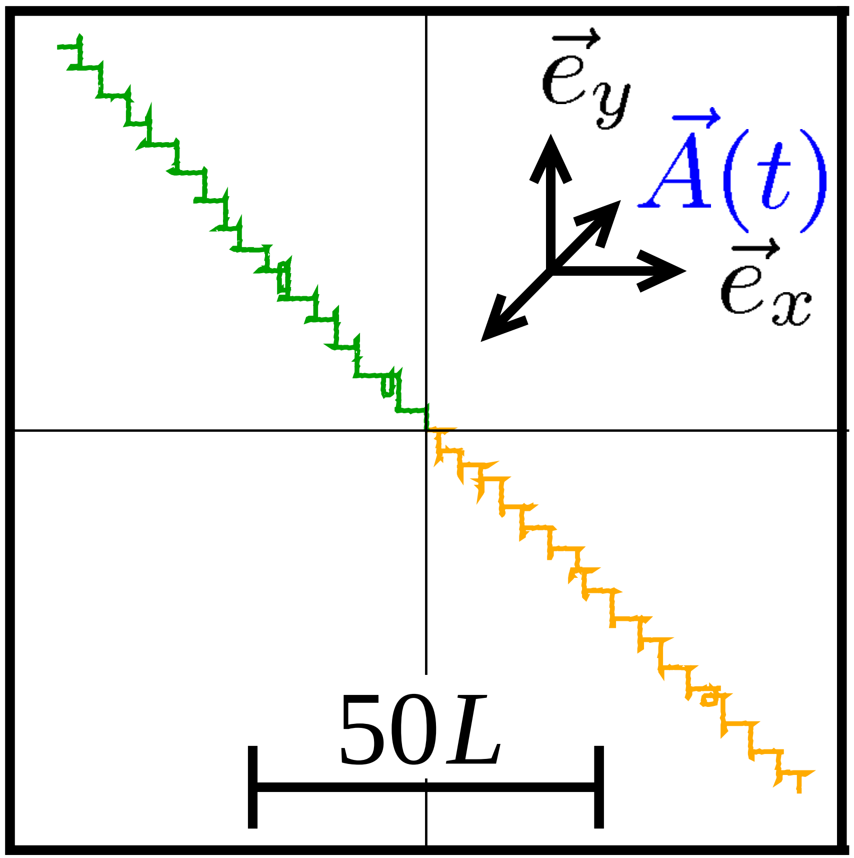
chiral partner: v identical, $\boxed{\vartheta \text{ symmetric about } 45^\circ}$ $(\vartheta_R = 90^\circ - \vartheta_L)$

Sorting Chiral Particles

[Speer, Eichhorn, Reimann, PRL **105**, 090602 (2010)]

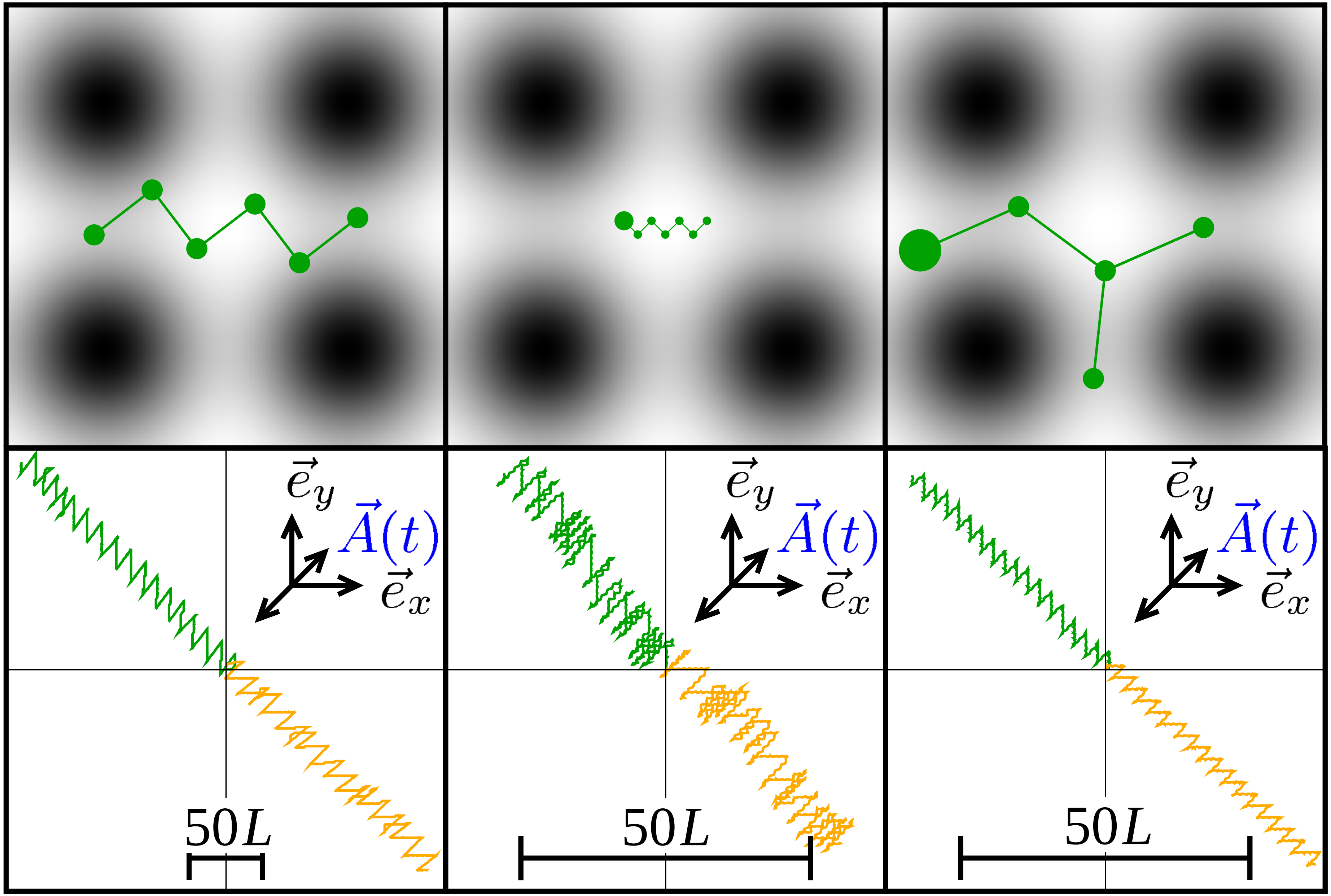
time-periodic $\vec{A}(t) = A(t)\,\vec{e}_\alpha$, $\alpha = 45°$, $A(t)$ alternating between 6 and -4





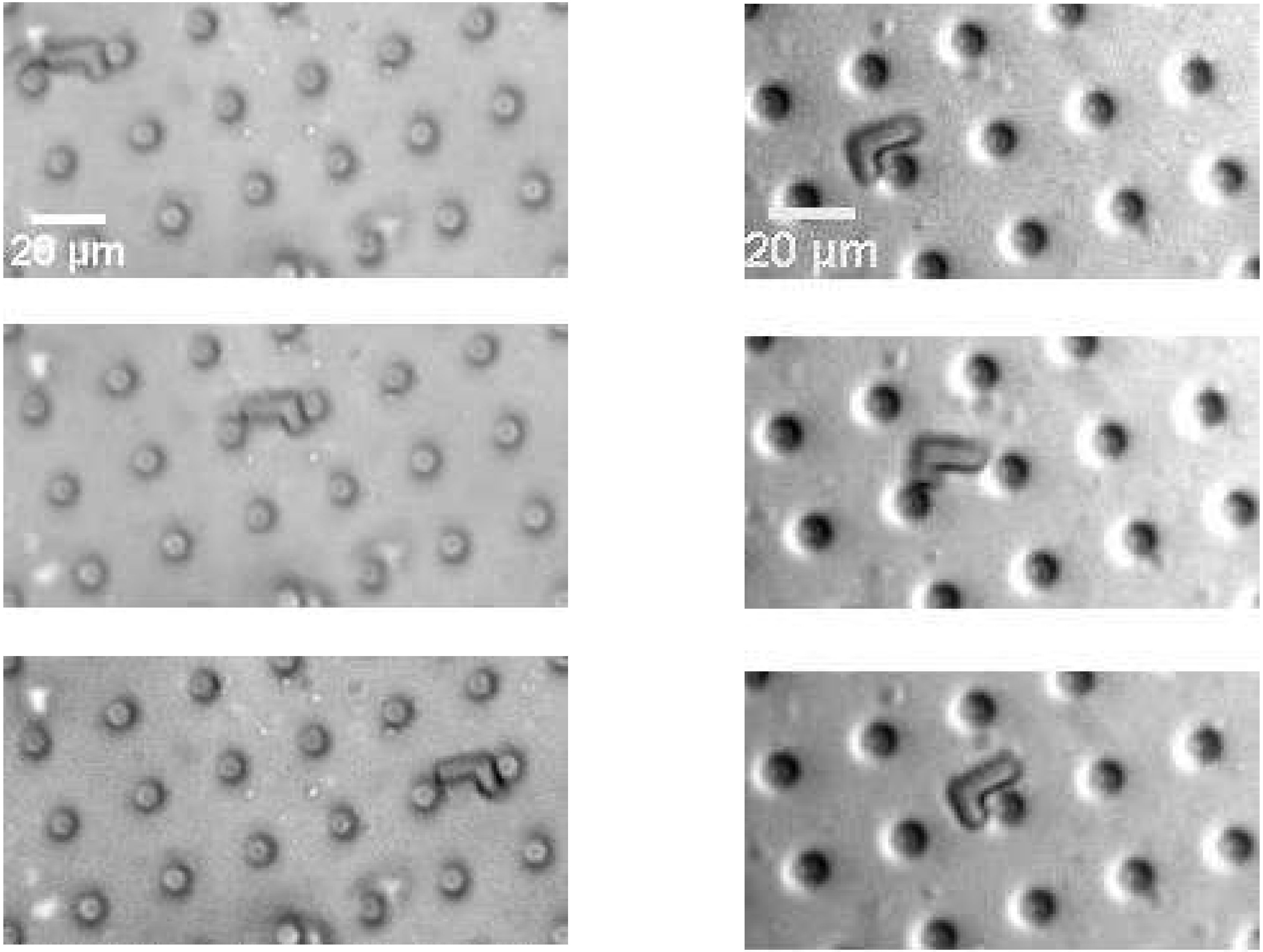
$\vec{r}(t)$ for $t \in [0, 100]$, $kT = 0.02$, $A(t) = 6$ for 2 time units, $A(t) = -4$ for 4 time units

Other chiral objects



First experimental steps

[Experiment: Wegener, Regtmeier, Anselmetti. Theory: Fliedner, Reimann]



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Peter Reimann
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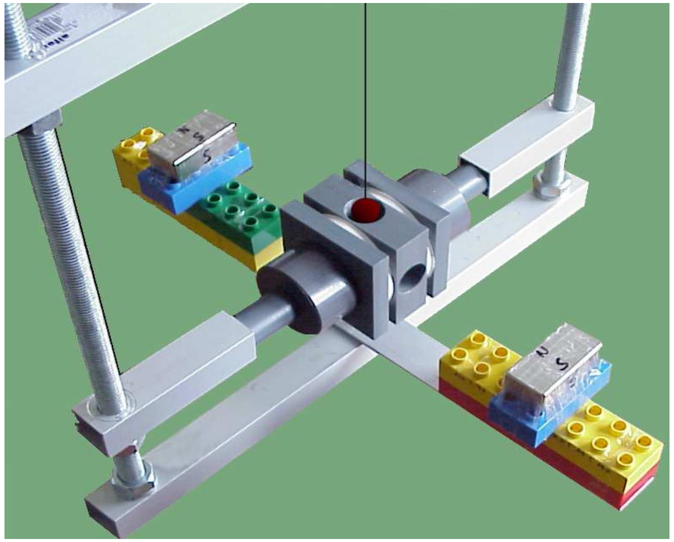
Ferrofluid Ratchet

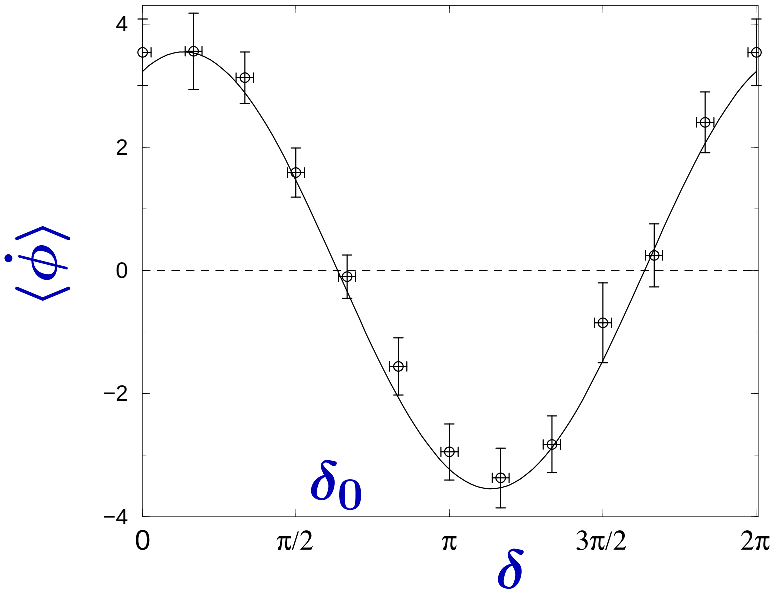


Theory: A. Engel, P. R.

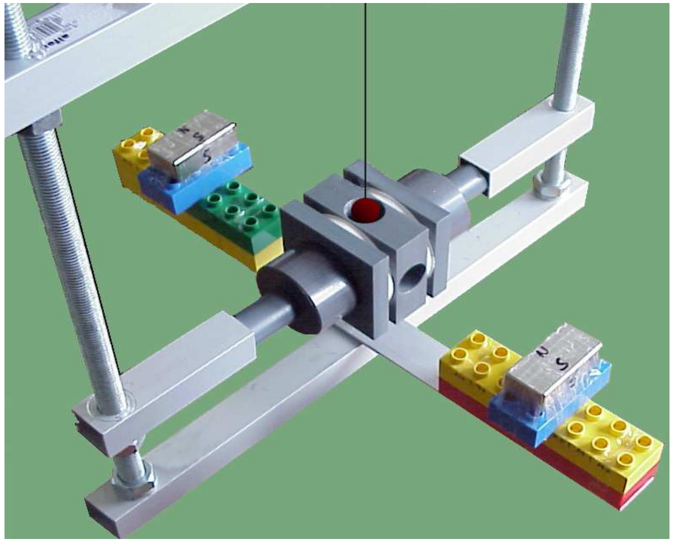
Experiment: H.-W. Müller, A. Jung

PRL **91**, 060602 (2003)

$$H_y(t) \propto \sin(\omega t) + \sin(2\,\omega t + \delta)$$



Ferrofluid Ratchet



Theory: A. Engel, P. R.

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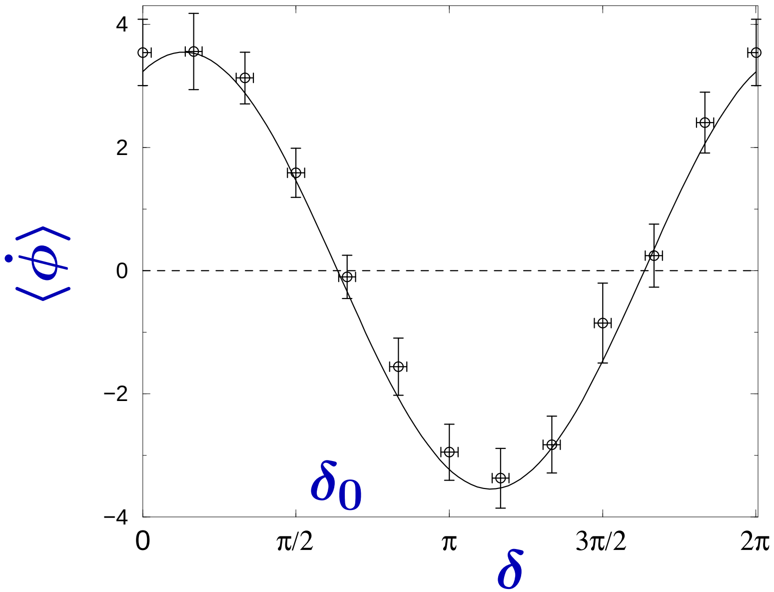

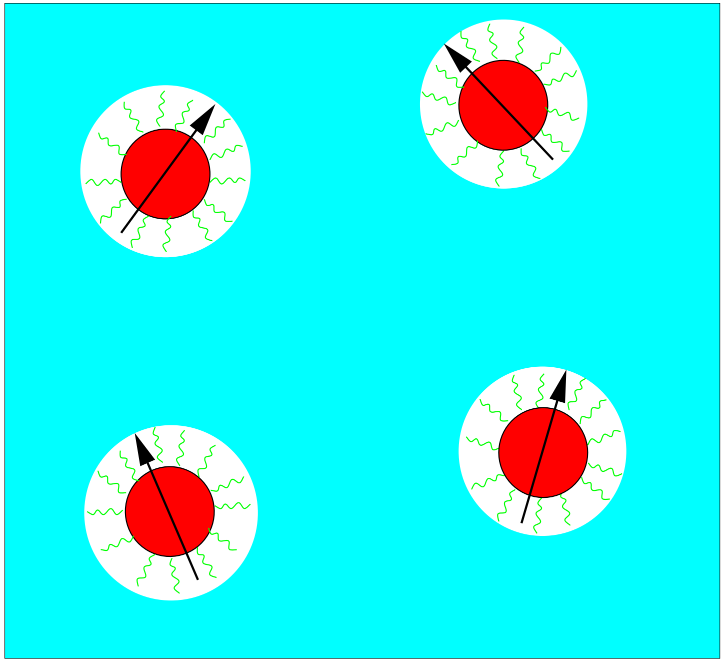

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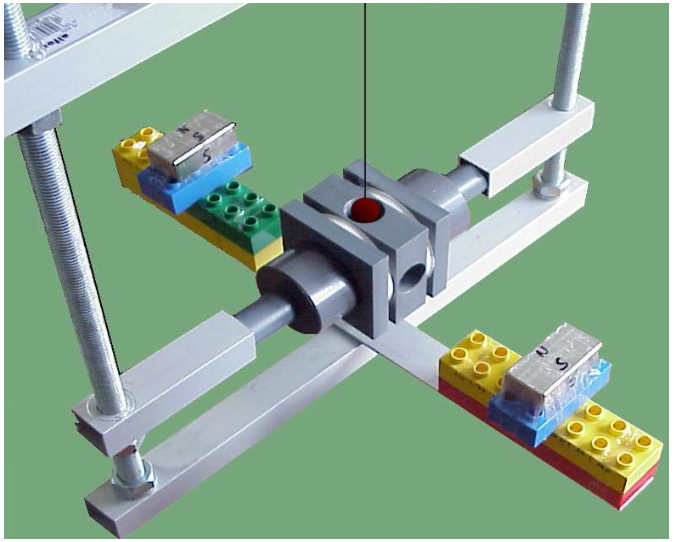

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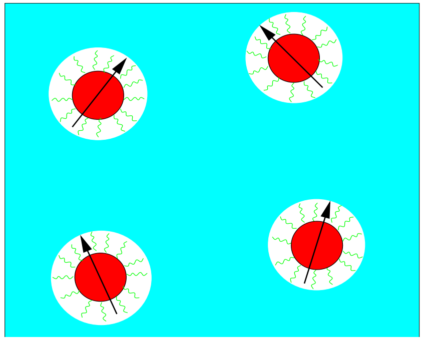

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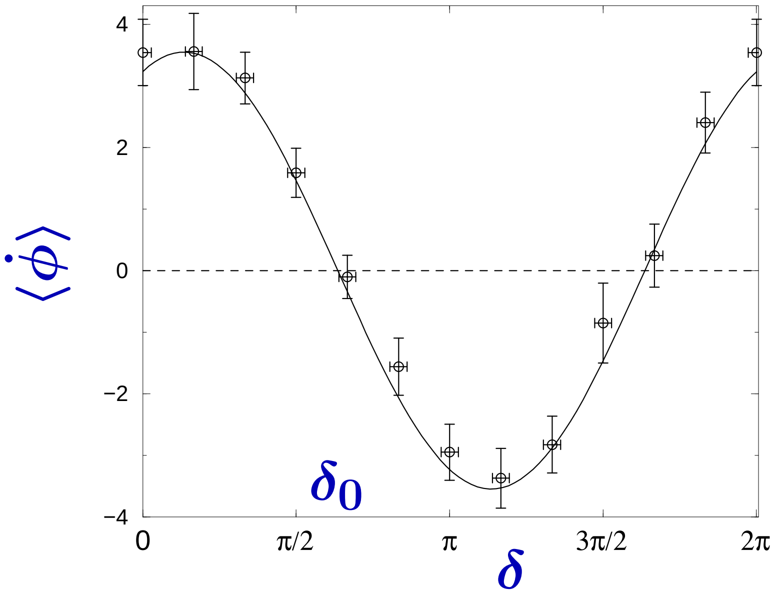

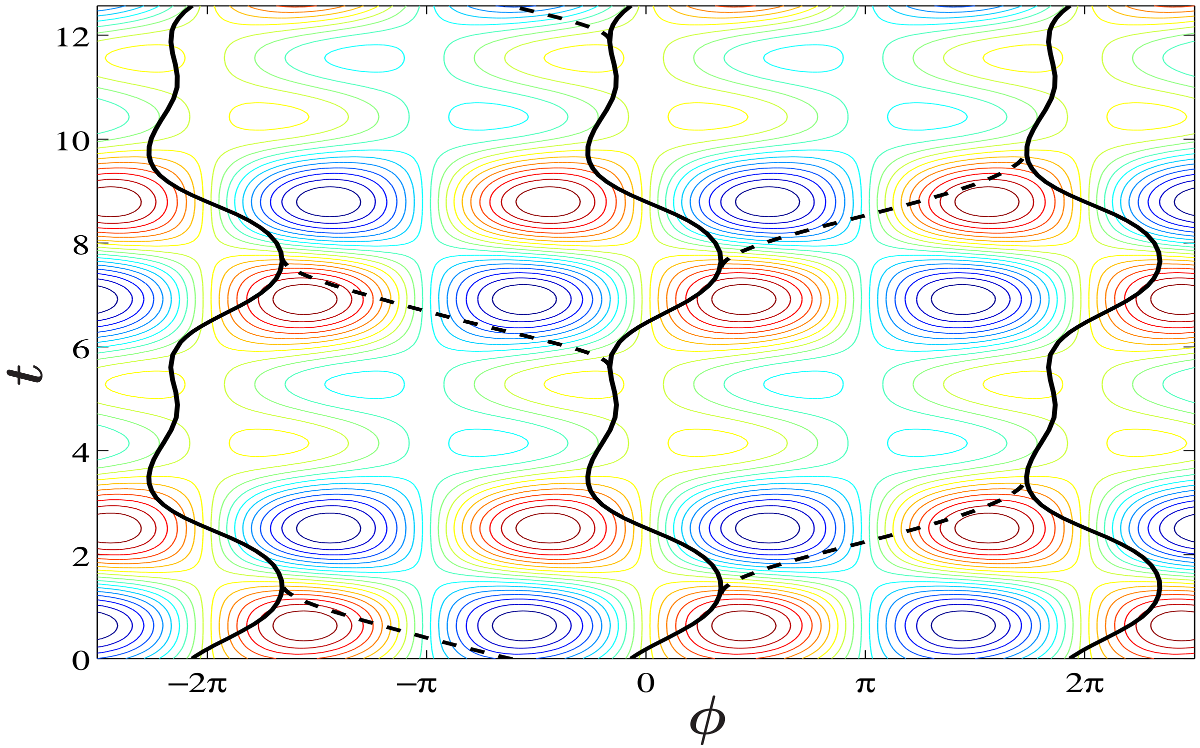

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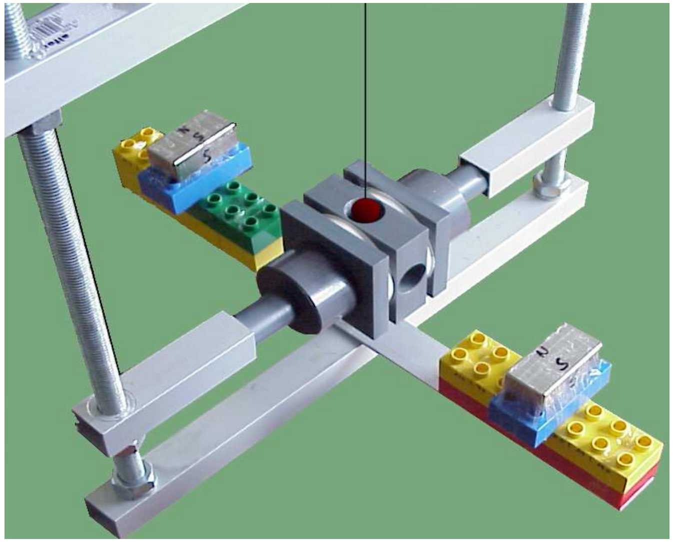

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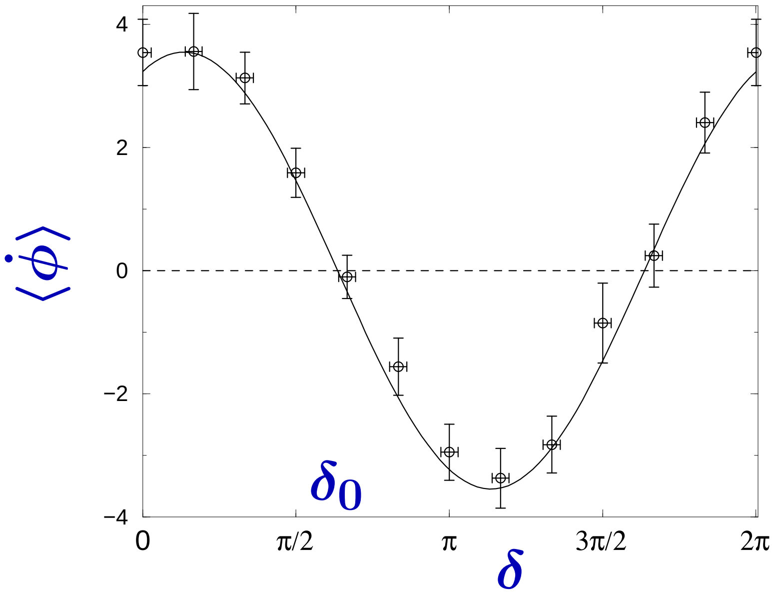

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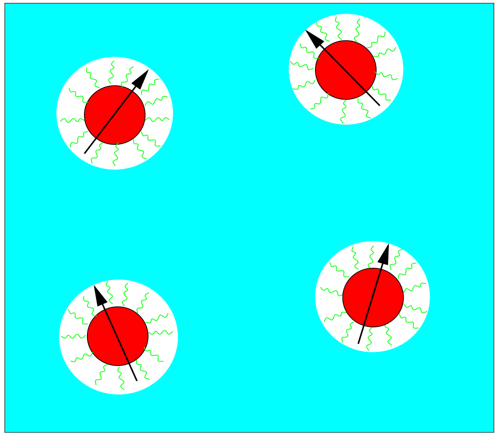


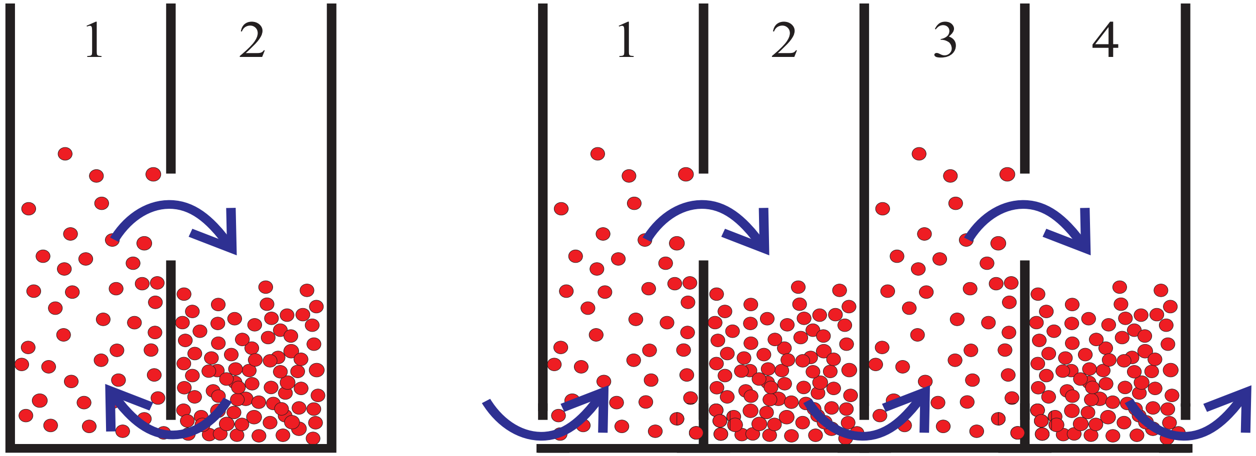
In the plane:

Supersymmetry

[P. R., PRL **86**, 4992 (2001)]

Granular Ratchet

[v.d. Meer, P. R., v.d. Weele, Lohse, PRL **92**, 184301 (2004); J. Stat. Mech. P07021 (2007)]



(a) (b)

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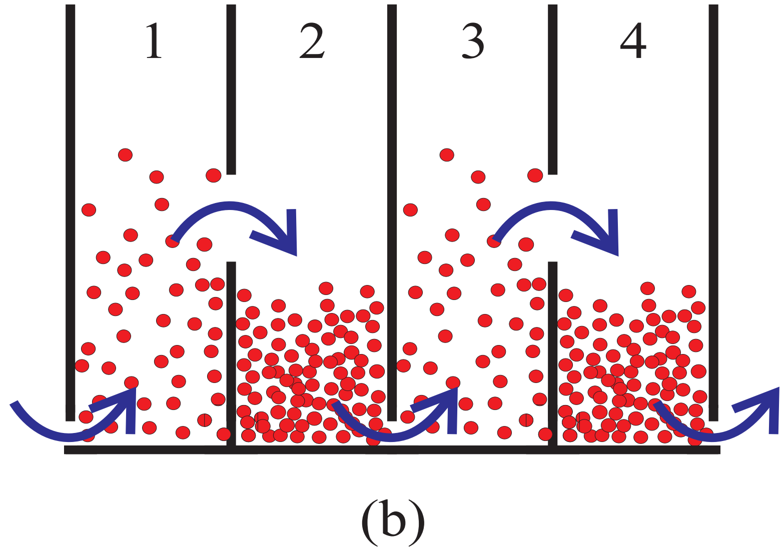
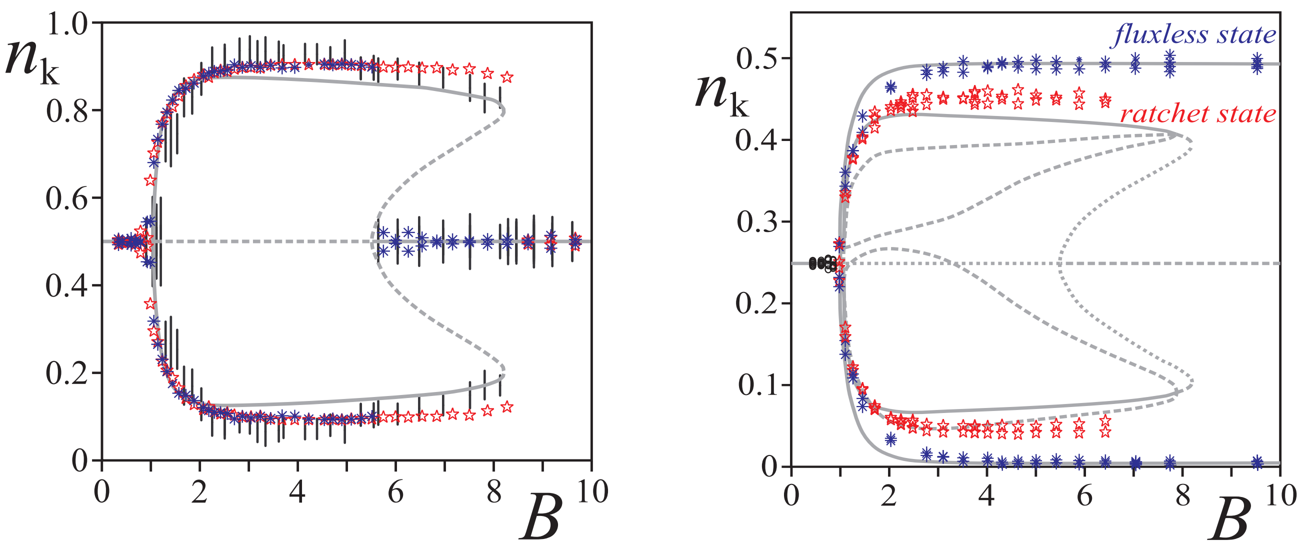
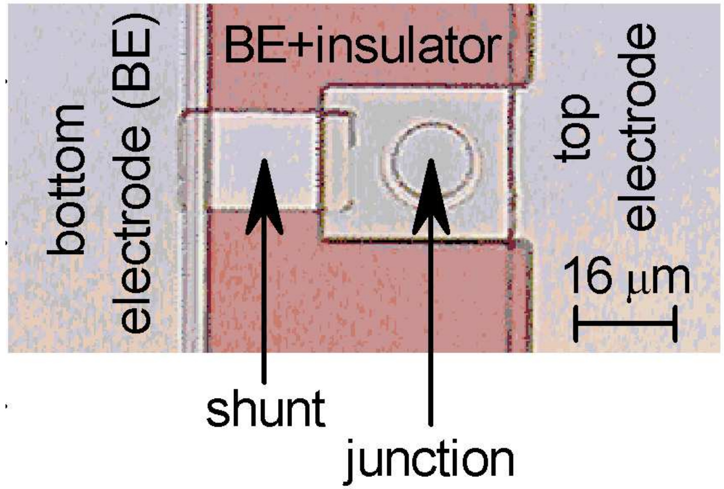
[v.d. Meer, P. R., v.d. Weele, Lohse, PRL **92**, 184301 (2004); J. Stat. Mech. P07021 (2007)]

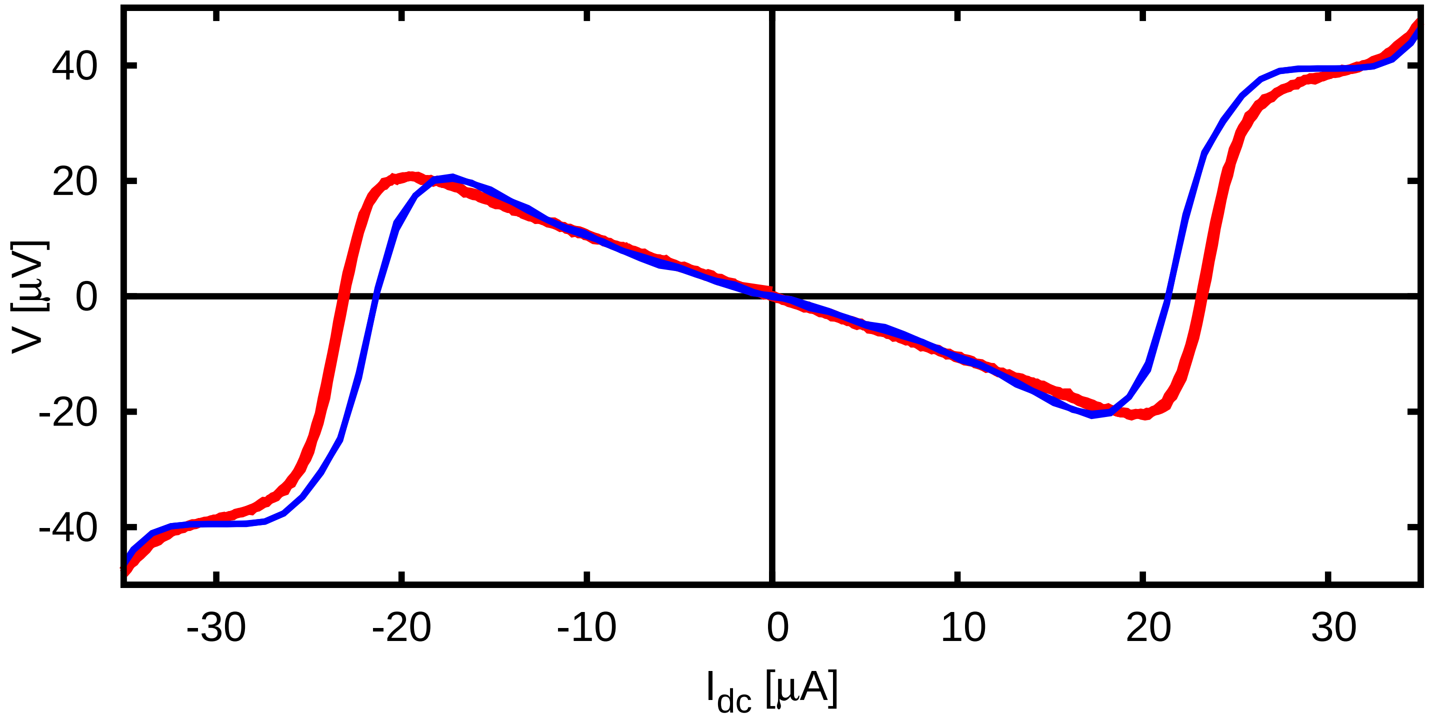




Negative Absolute Resistance in a Josephson Junction

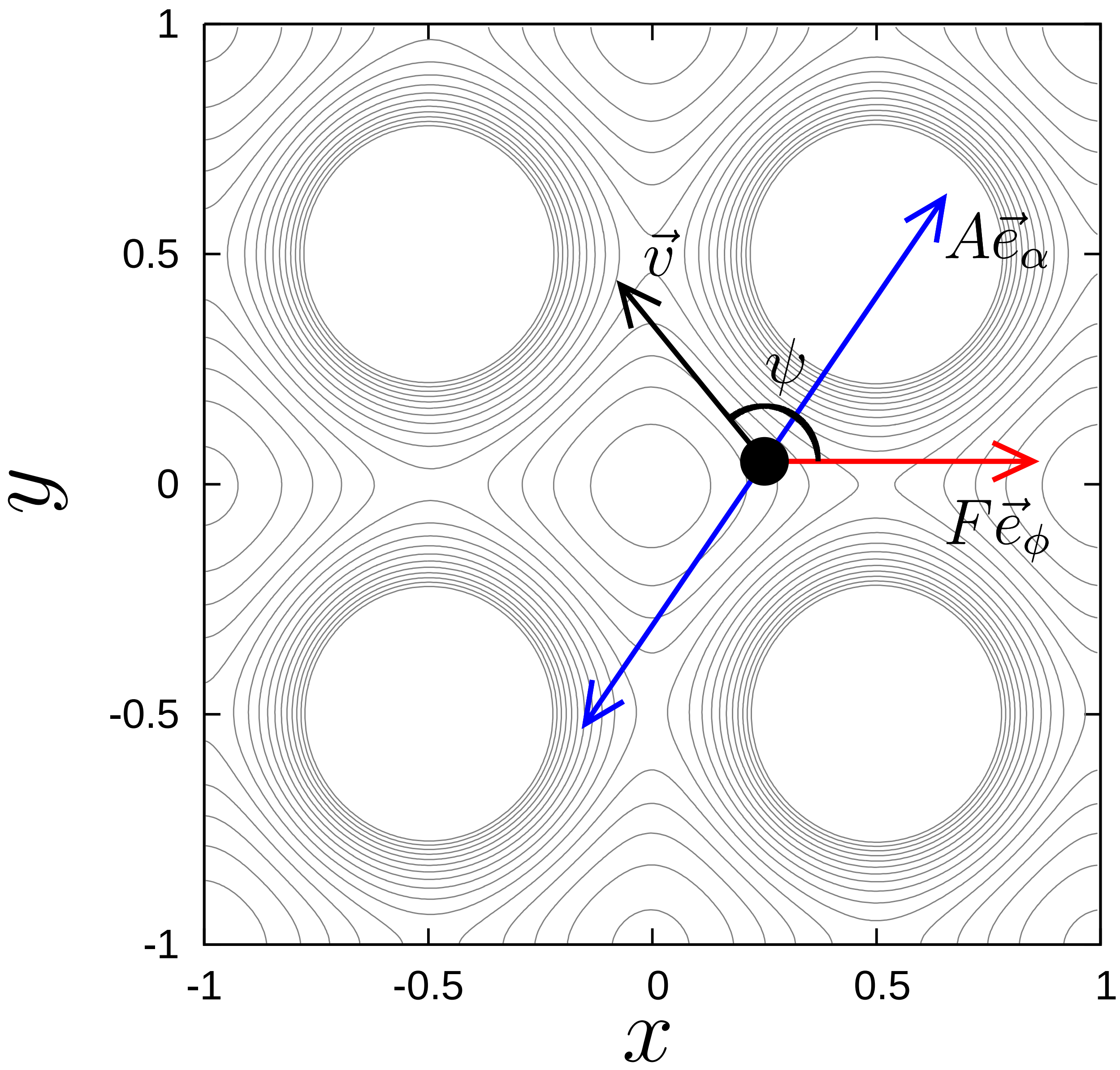
J. Nagel, D. Speer, T. Gaber, A. Sterck, R. Eichhorn, P. Reimann,
K. Ilin, M. Siegel, D. Koelle, R. Kleiner, Phys. Rev. Lett. 100, 217001 (2008)





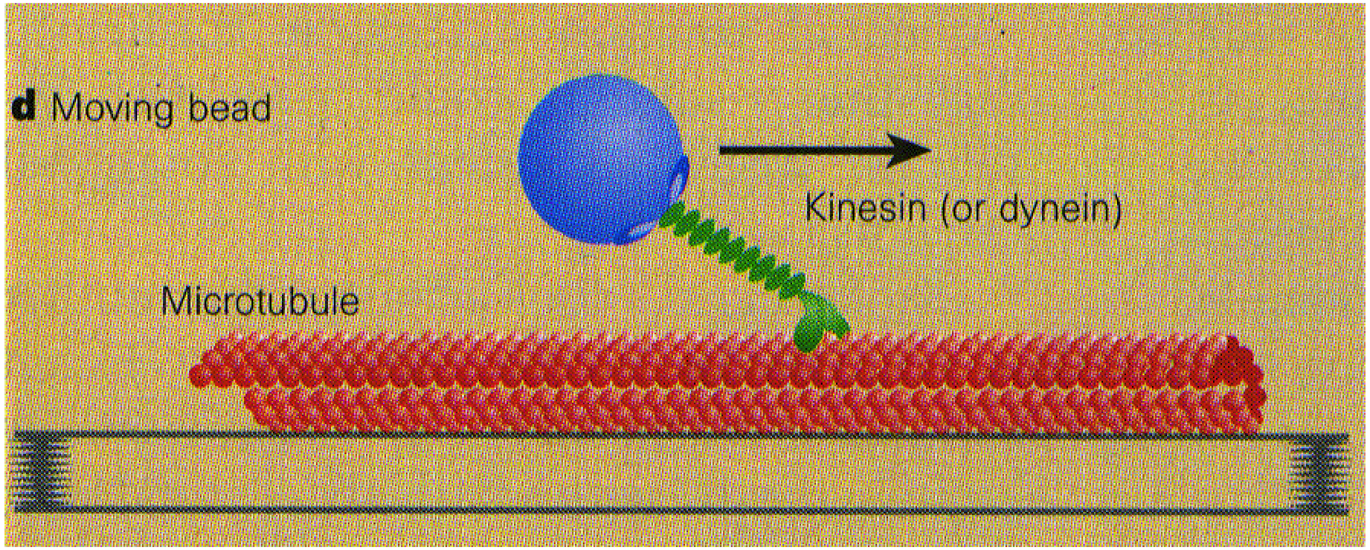
Directing Brownian Motion on a Periodic Surface

D. Speer, R. Eichhorn, P. Reimann, Phys. Rev. Lett. 102, 124101 (2009)



Molecular Motors

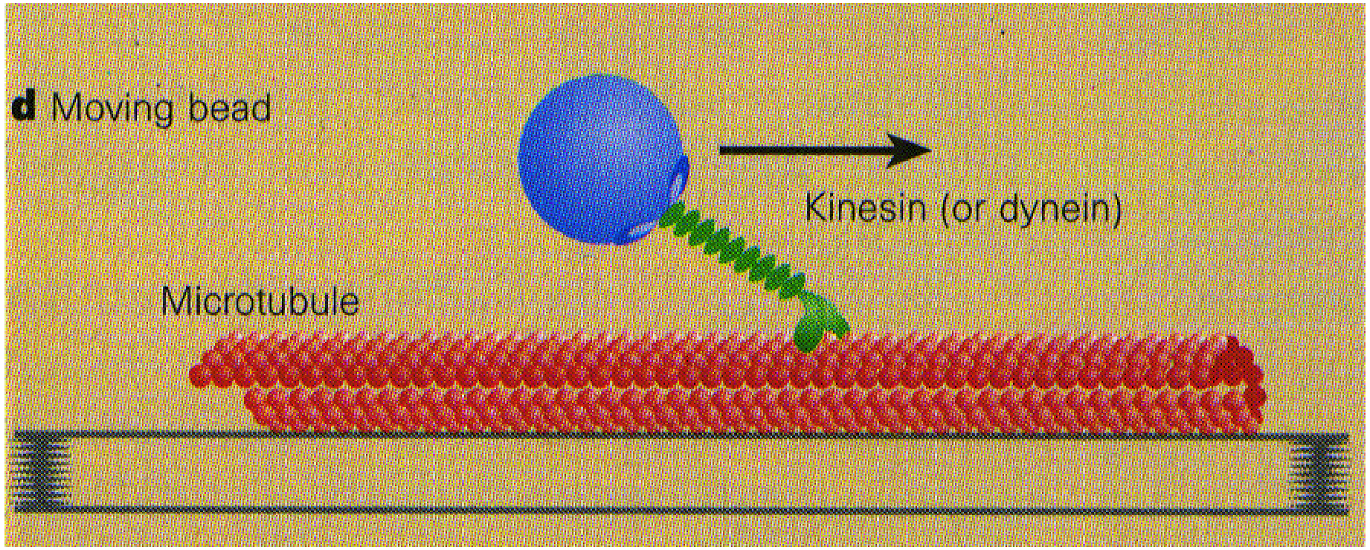
[Huxley 1957, Vale and Oosawa 1990, ...]

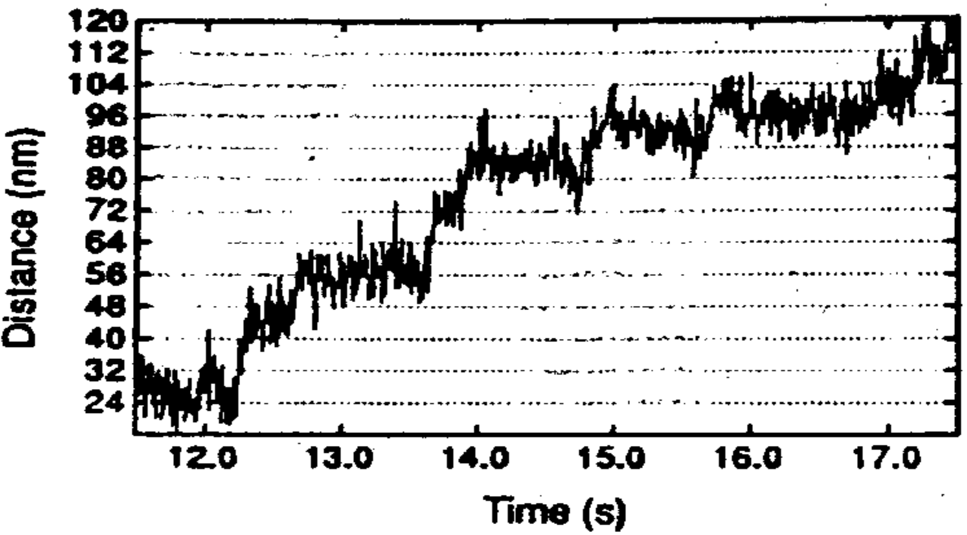


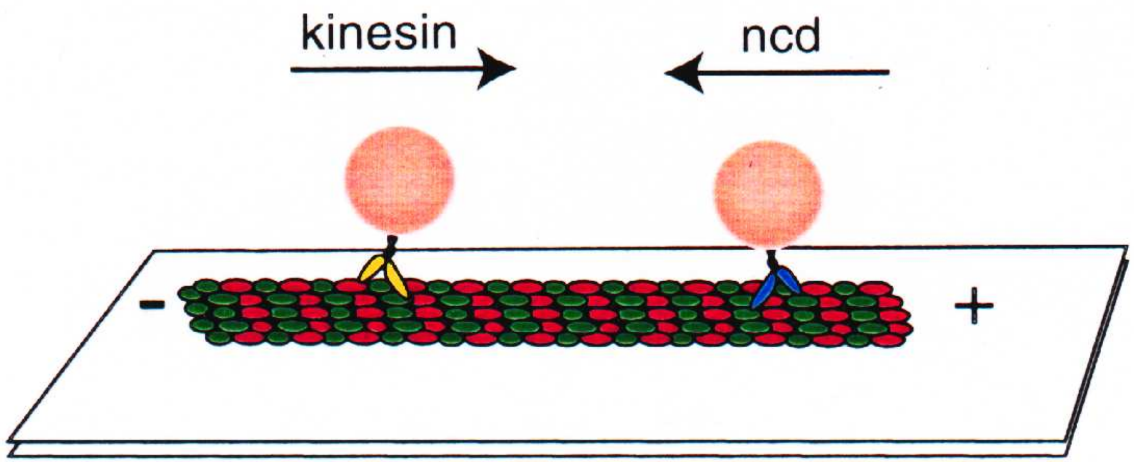
- $x(t)$: mech./geom. configuration (cyclic) or position
- $V(x)$ periodic & asymmetric
- length scale $\sim$ 10nm $\Rightarrow$ **thermal noise** relevant
- Chemical reaction cycle (ATP-hydrolysis) $\Rightarrow$ heat production
  $\Rightarrow$ local temperature changes $T(t)$

Molecular Motors

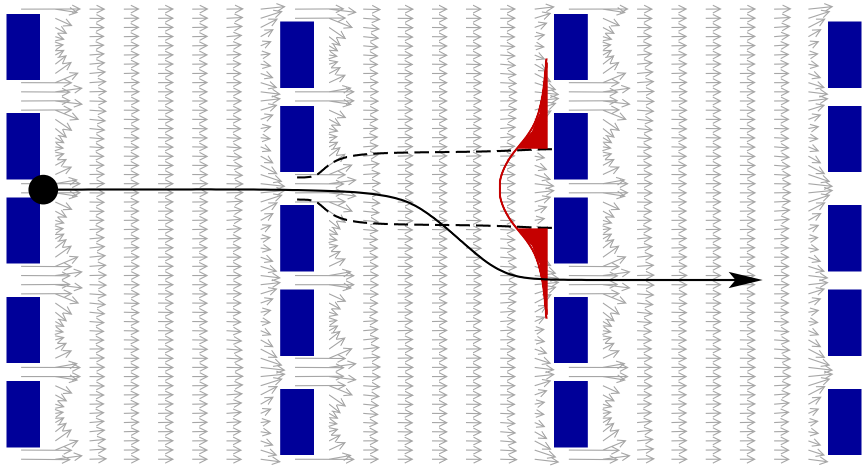
[Huxley 1957, Vale and Oosawa 1990, ...]



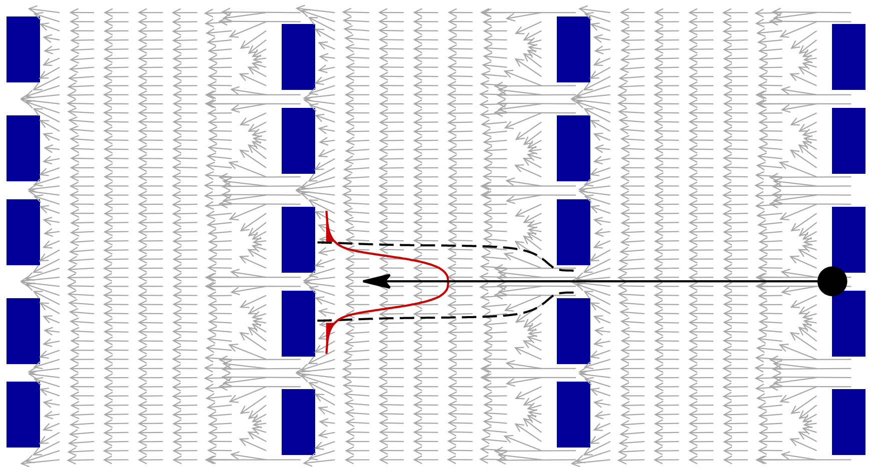




Physical Mechanism



$U_{DC} < 0\,, \quad U_{DC} + U_0 > 0$

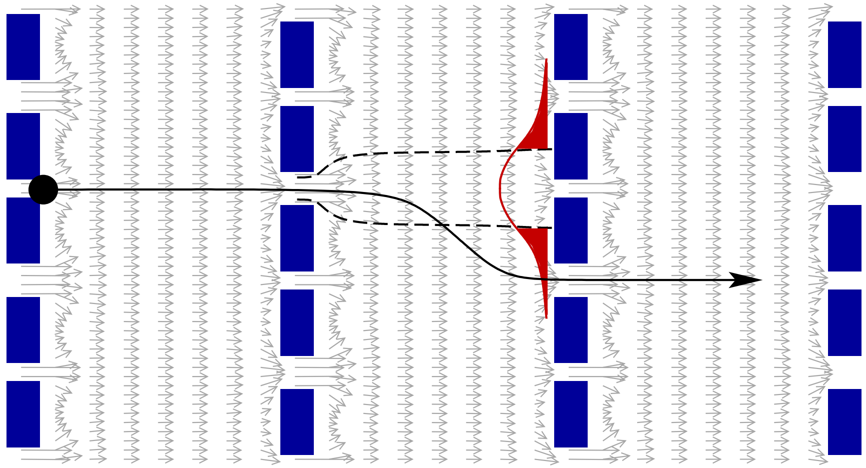


$U_{DC} < 0\,, \quad U_{DC} - U_0 < 0$

Indispensable:

- **"particle traps"**
- **fluctuations (diffusion)**

Physical Mechanism



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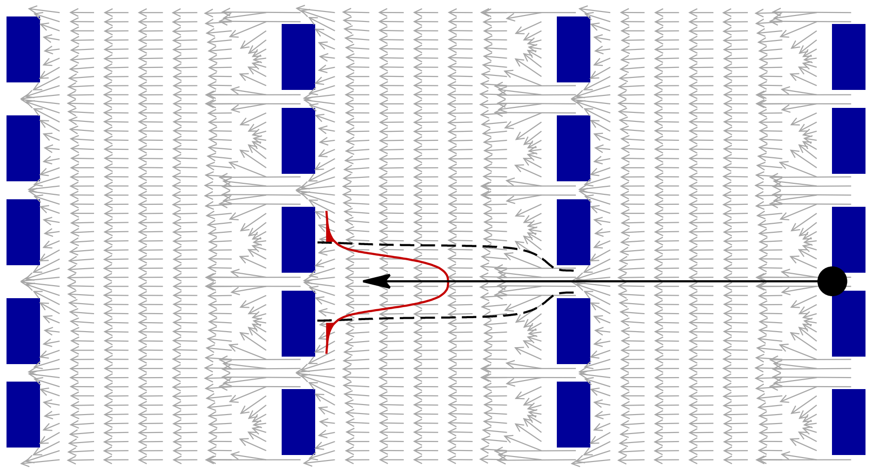

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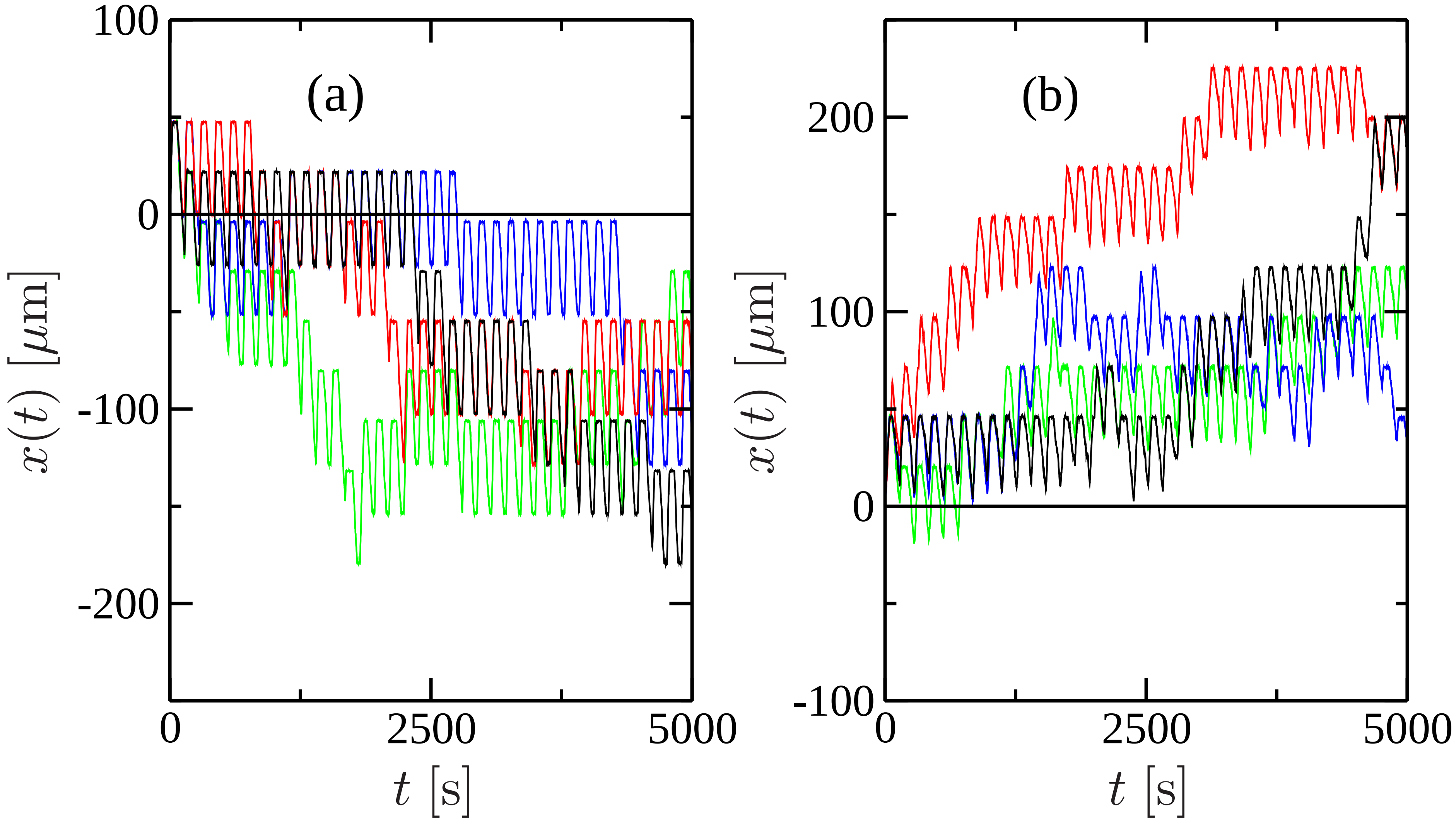
Theoretical concept: Phys. Rev. Lett. **88**, 190601 (2002)

Experiment versus theory: Nature **436**, 928 (2005)

Single Particle Trajectories

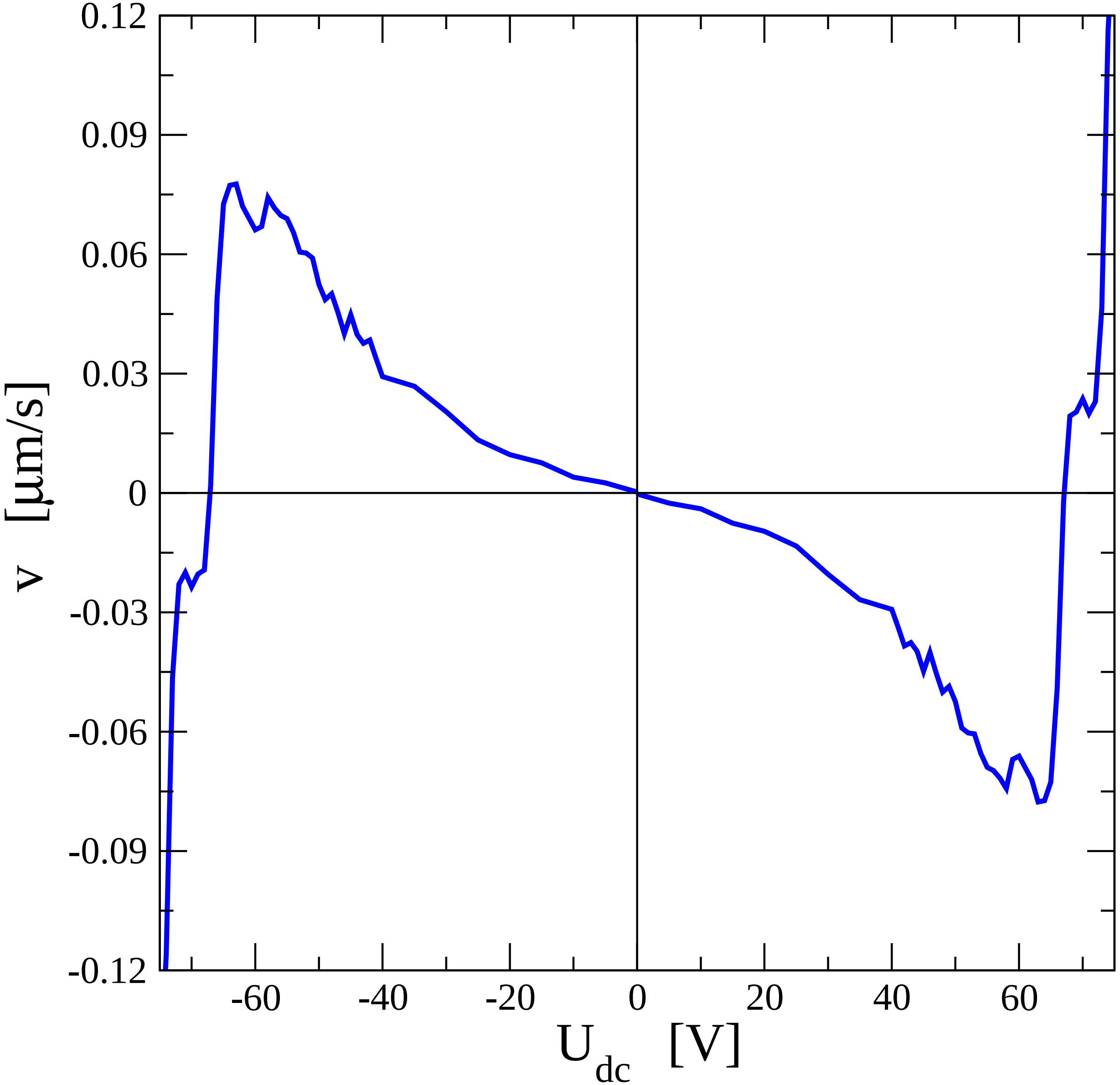
($U_0 = 6$ V, $\tau = 70$ s, $U_{DC} = 2$ V)

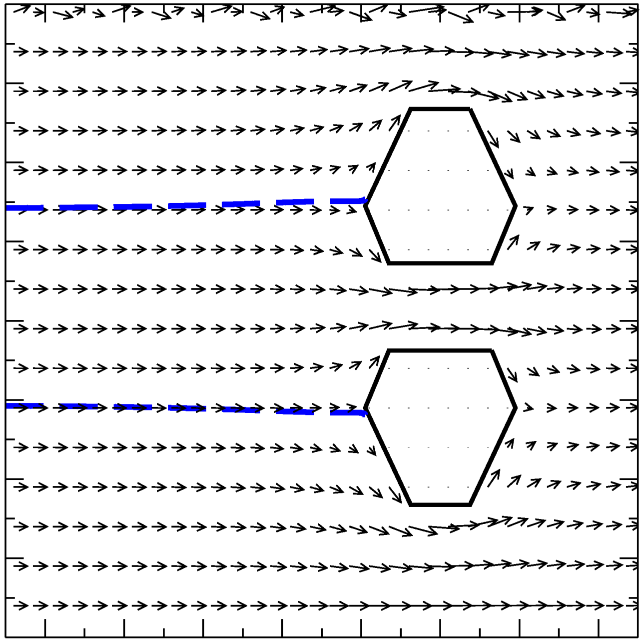
1.9 μm particles **2.8 μm particles**



Optimized Microstructure

[Regtmeier, Grauwin, Eichhorn, Reimann, Anselmetti, Ros, J. Sep. Sci. **30**, 1461 (2007)]





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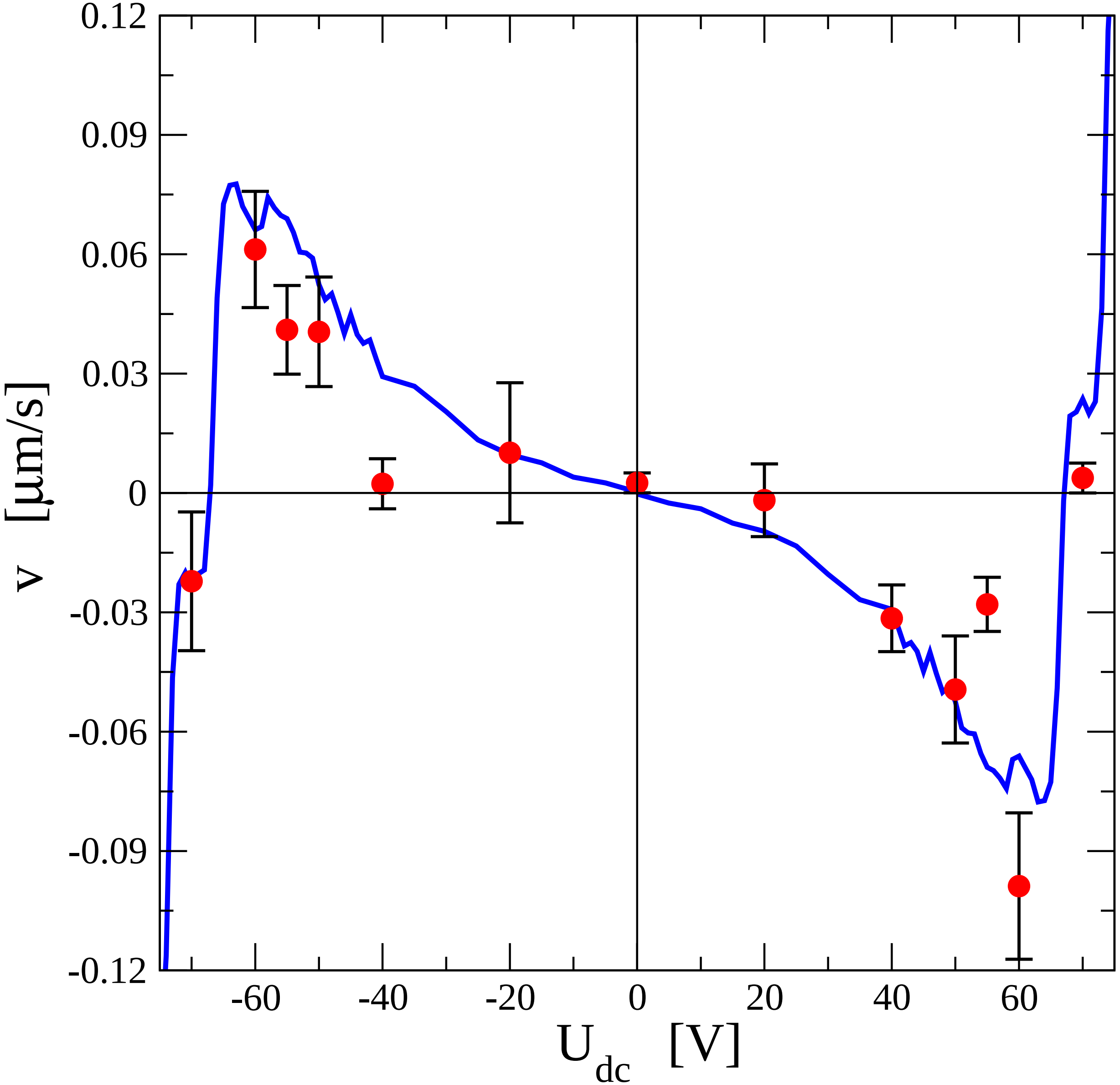

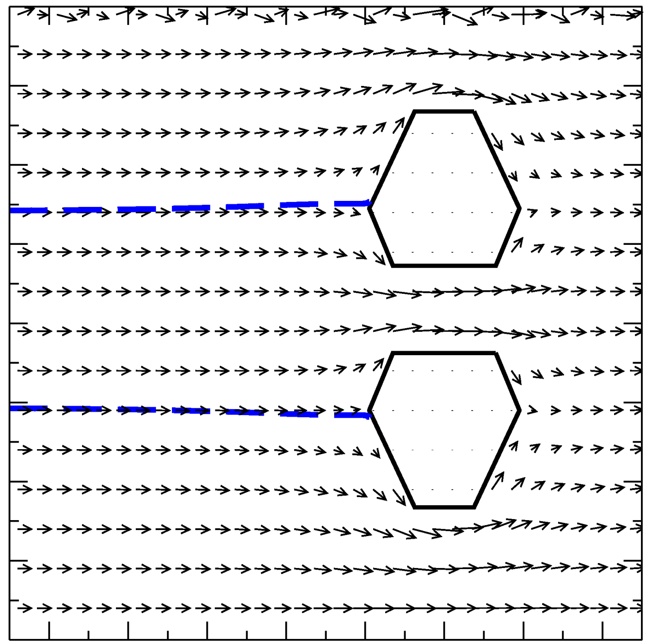

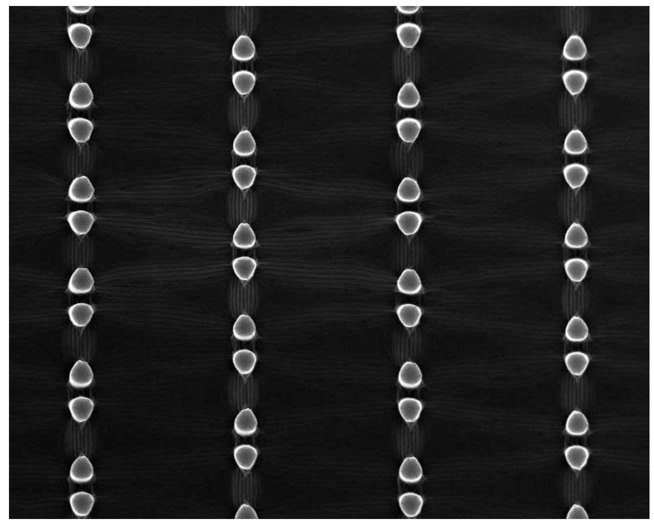

Boundary Conditions

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fluid (buffer): conductor, $\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}) \;||\; \vec{\boldsymbol{E}}(\vec{\boldsymbol{r}})$, e.g. $\vec{j} = \sigma\vec{E}$

solid (PDMS): insulator, $\vec{\boldsymbol{j}}(\vec{\boldsymbol{r}}) = \vec{\mathbf{0}}$

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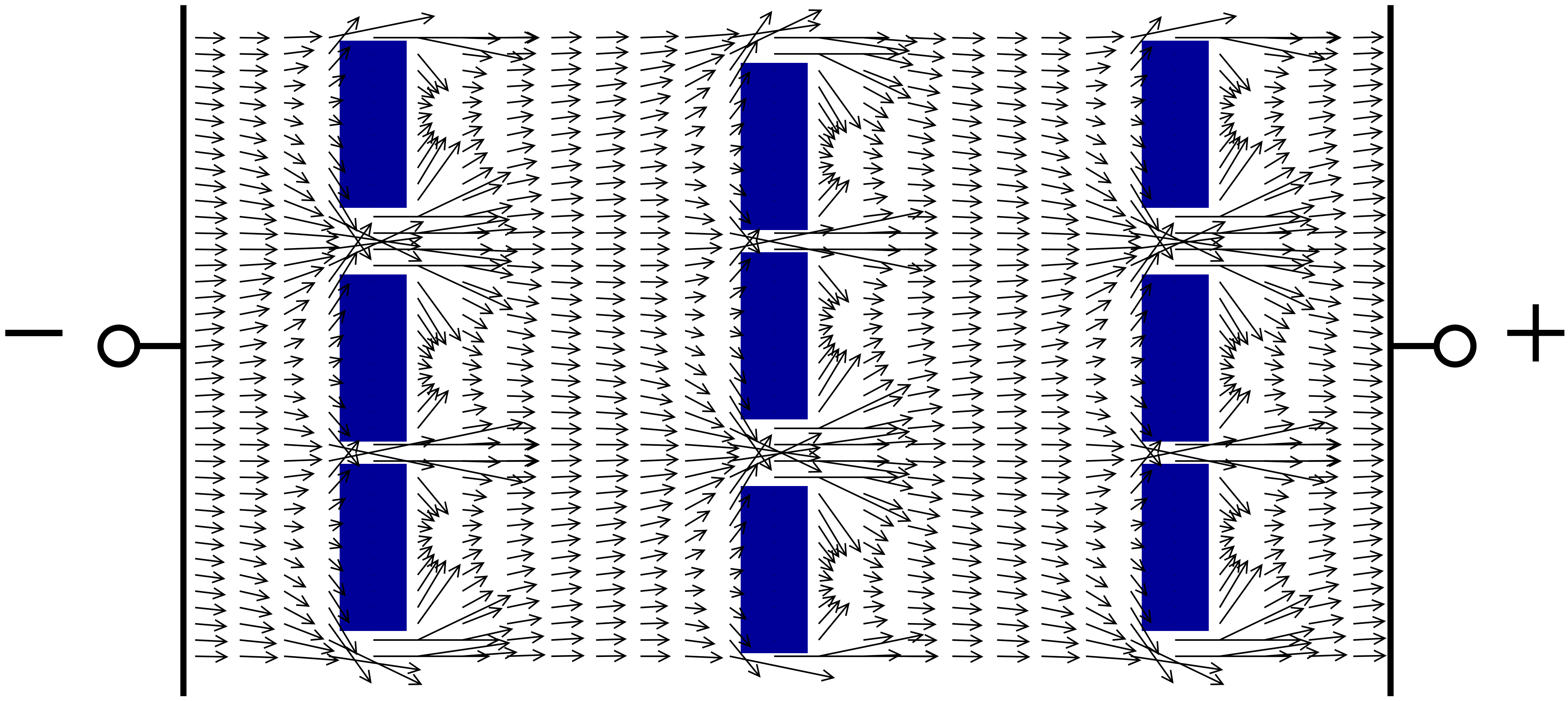
$\Rightarrow$ $\boldsymbol{\vec{E}_{\perp}(\vec{r}) = \vec{n}(\vec{r})\cdot\nabla\phi(\vec{r}) = \vec{0}}$ **at border** (Neumann b.c.)

$\Rightarrow$ no particle trap at border possible

Earnshaw's Theorem: no trap inside fluid ($\nabla\vec{E} = \rho/\epsilon = 0$)

Electrical Field

$\Delta\phi(\vec{r}) = 0$ with mixed boundary conditions $\Rightarrow$ $\vec{\boldsymbol{E}}(\vec{\boldsymbol{r}}) = -\boldsymbol{\nabla}\phi(\vec{\boldsymbol{r}})$



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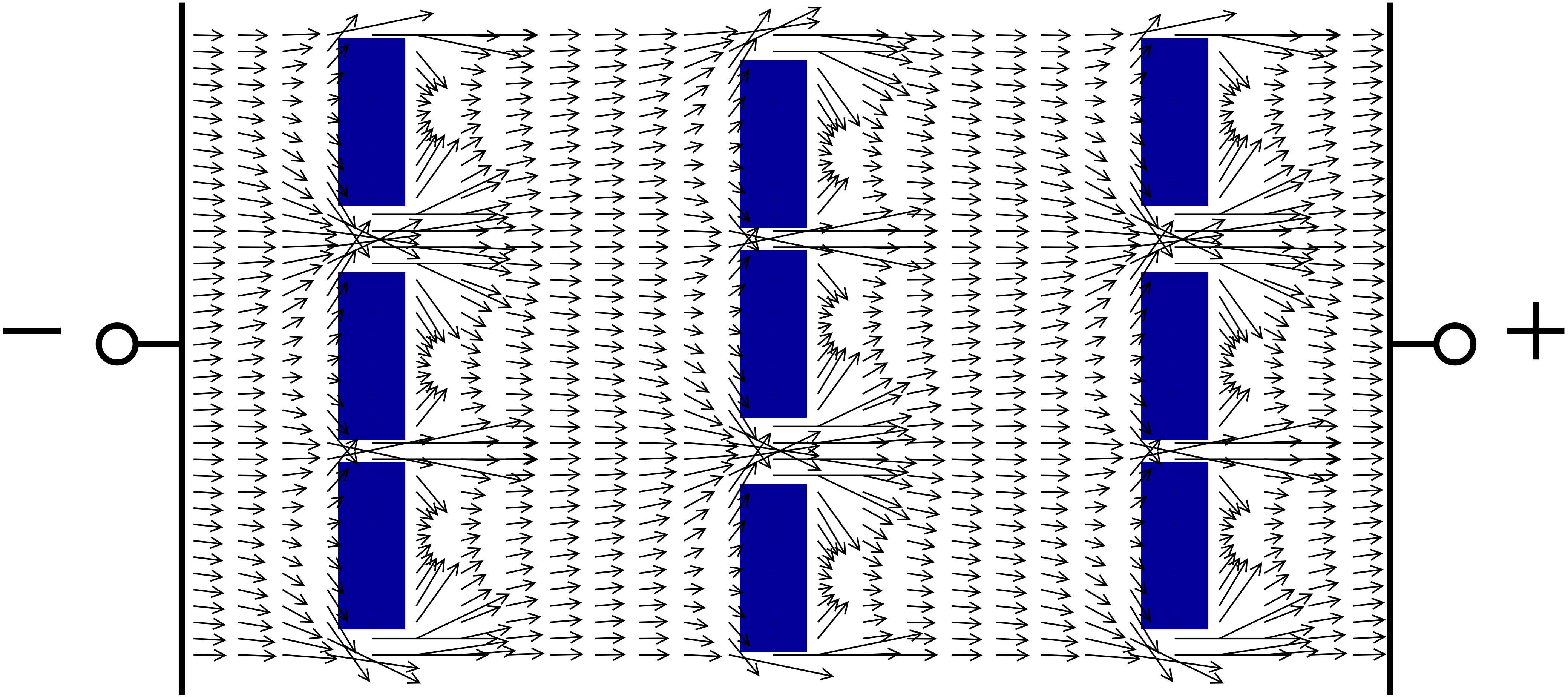

z-direction trivial

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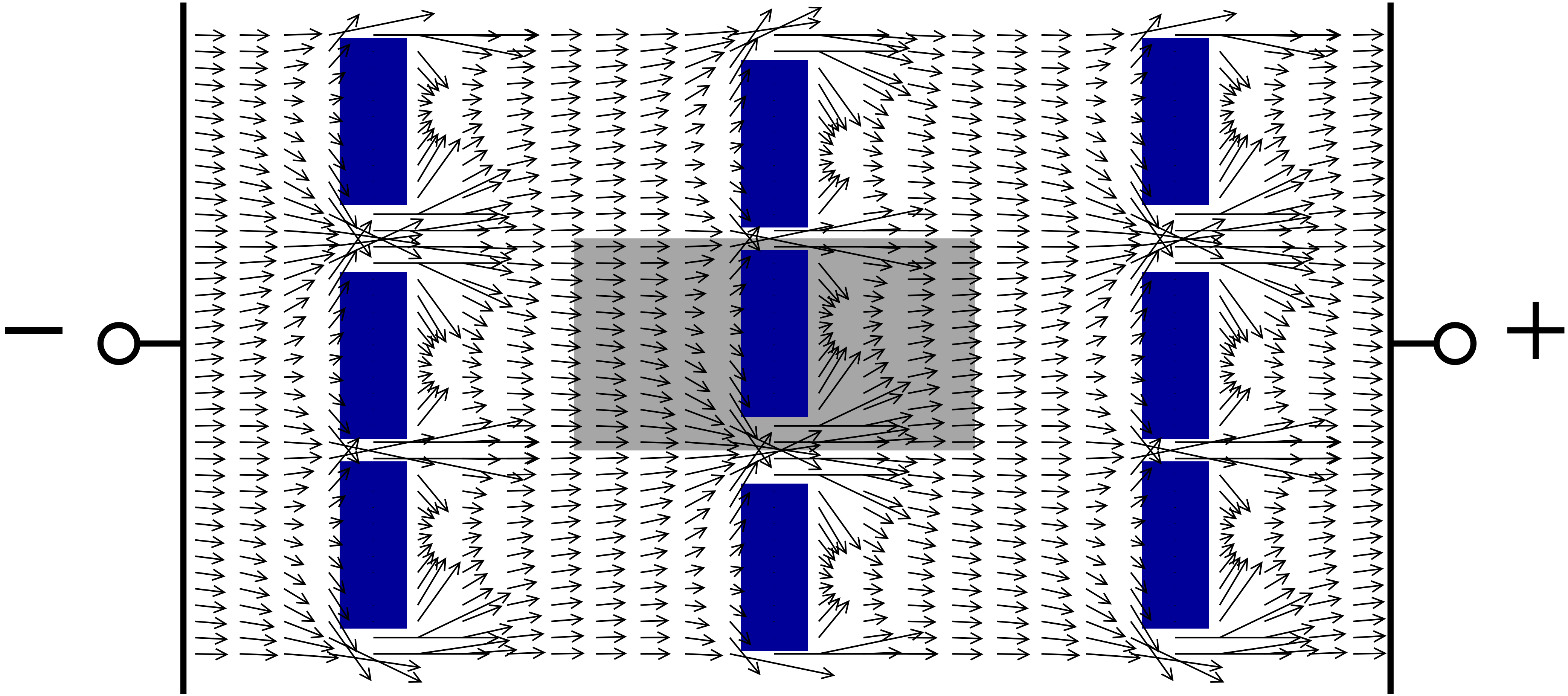
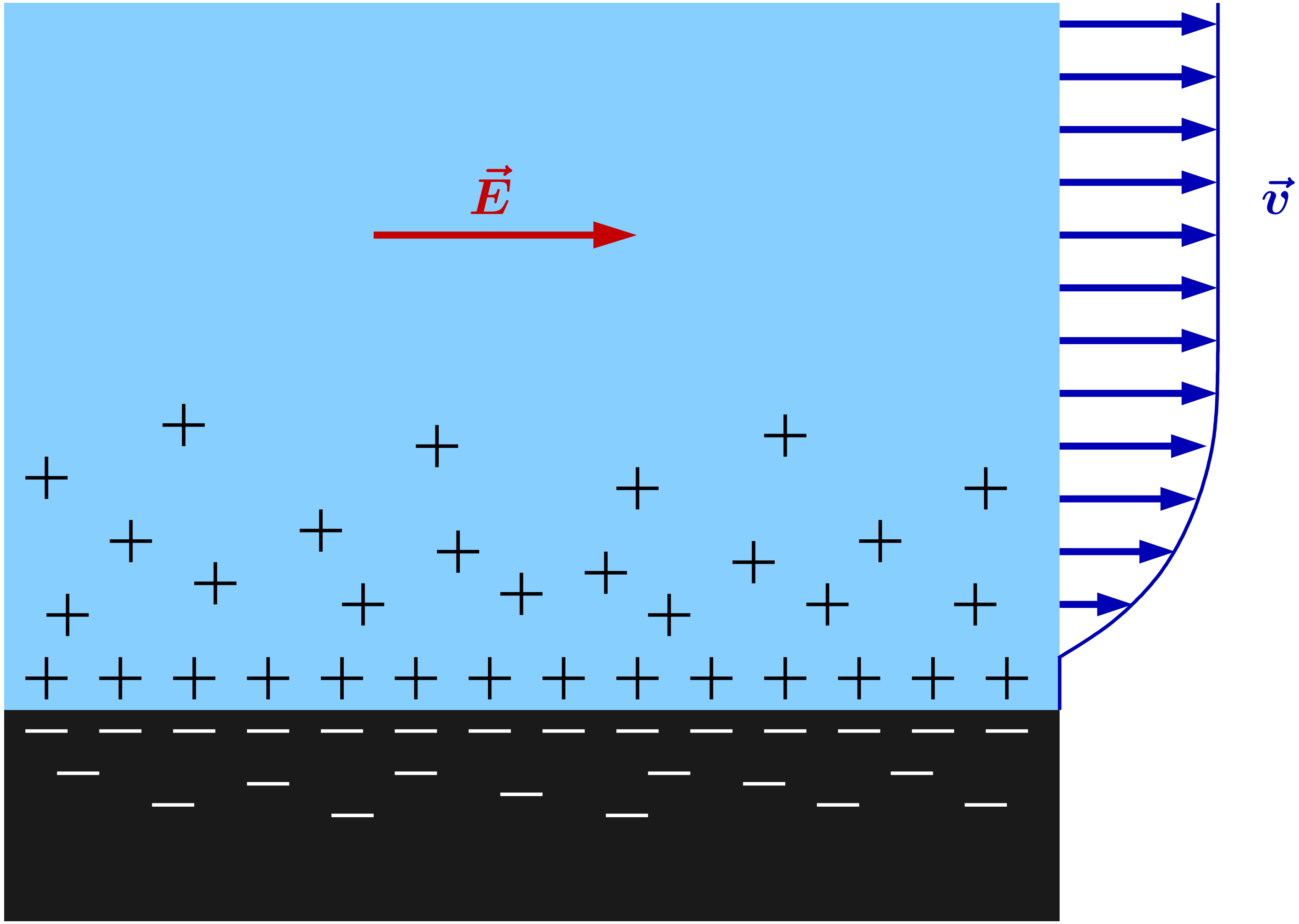


z-direction trivial

Central "unit cell" **periodically continued**

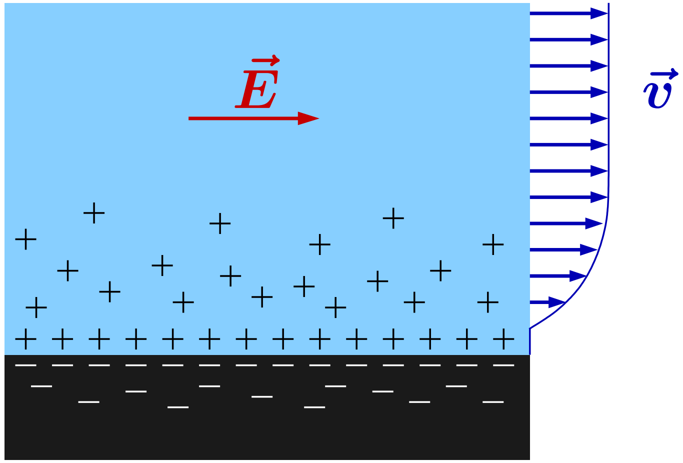
Electroosmosis



$$\vec{v}(\vec{r}) = \lambda \, \vec{E}(\vec{r})$$

$$\lambda = -\frac{\epsilon \, \zeta}{\nu}$$

Electroosmosis



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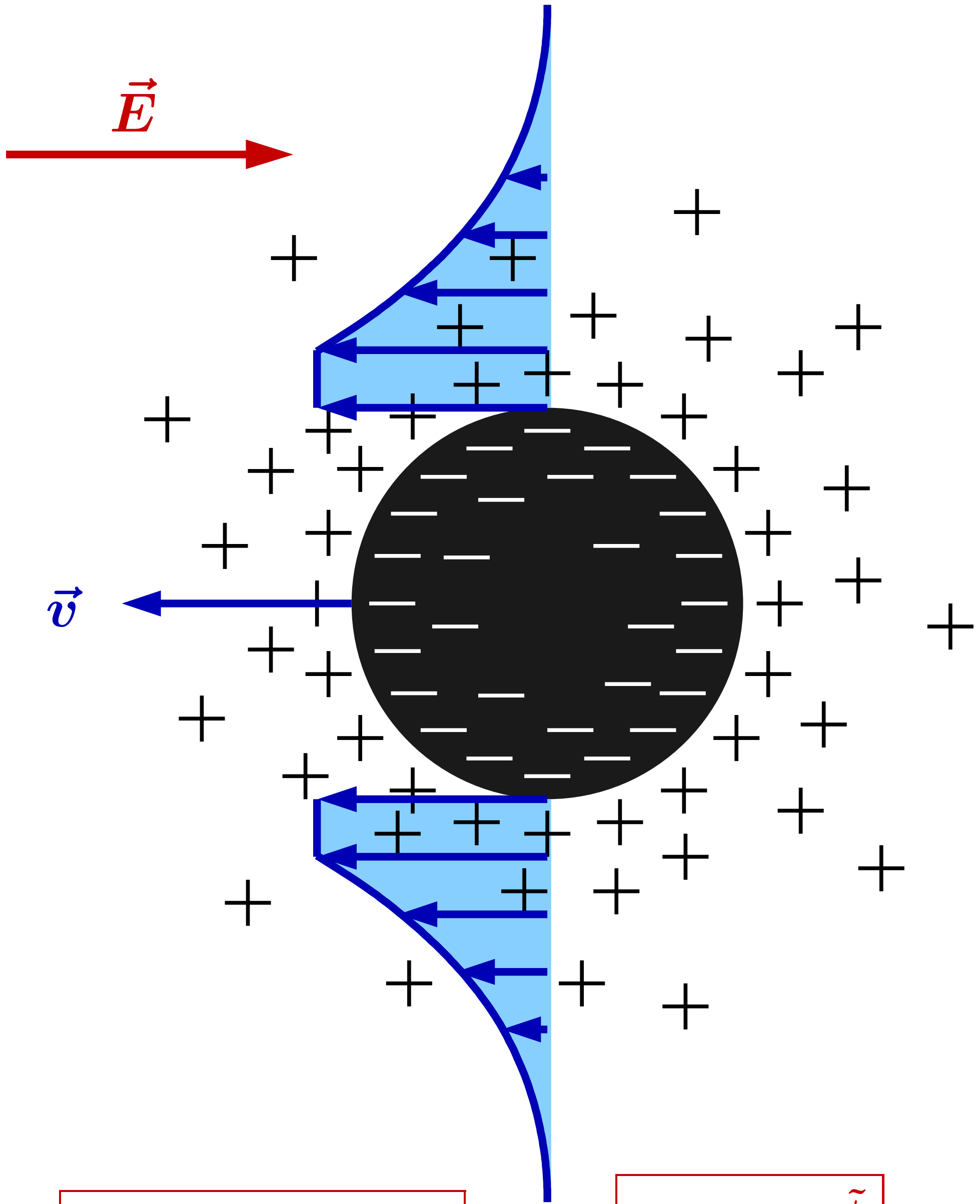
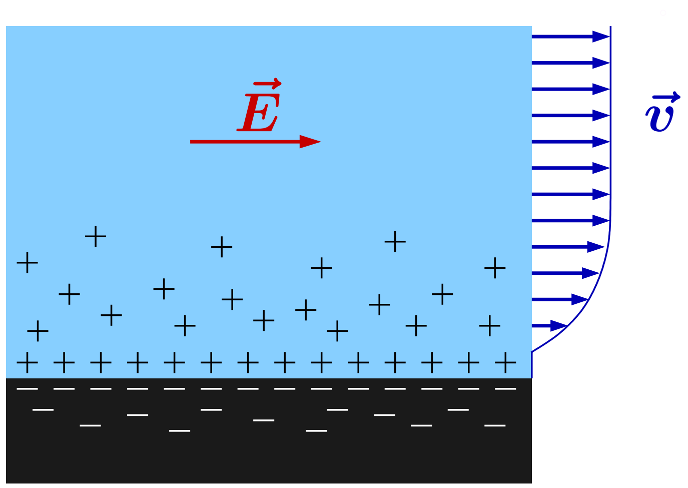
Electrophoresis



$$\boldsymbol{\vec{v}(\vec{r}) = \tilde{\lambda}\,\vec{E}(\vec{r})} \qquad \boldsymbol{\tilde{\lambda} = +\frac{\epsilon\,\tilde{\zeta}}{\nu}}$$

(no form factor, Smoluchowski 1903)

Electroosmosis



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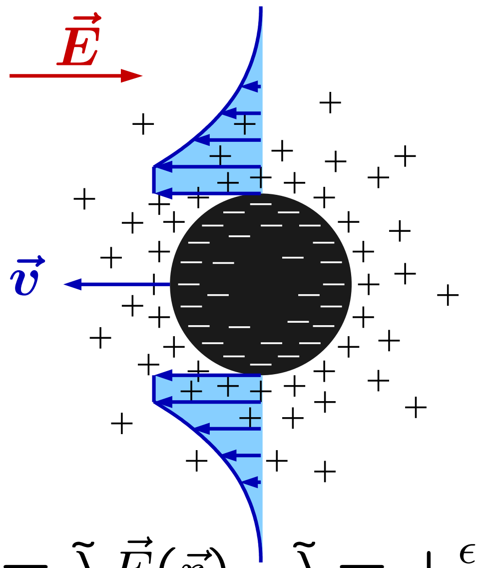
Electrophoresis

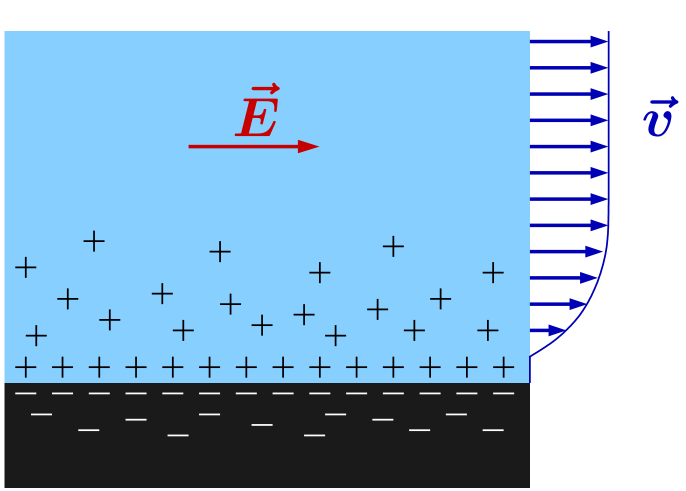


$$\vec{v}(\vec{r}) = \tilde{\lambda}\,\vec{E}(\vec{r})\,,\;\; \tilde{\lambda} = +\frac{\epsilon\,\tilde{\zeta}}{\nu}$$

$\Rightarrow$ net particle velocity

$$\boldsymbol{\vec{v}_0(\vec{r}) = (\tilde{\lambda} + \lambda)\,\vec{E}(\vec{r}) = \frac{\epsilon\,(\tilde{\zeta}-\zeta)}{\nu}\,\vec{E}(\vec{r})}$$

Electroosmosis



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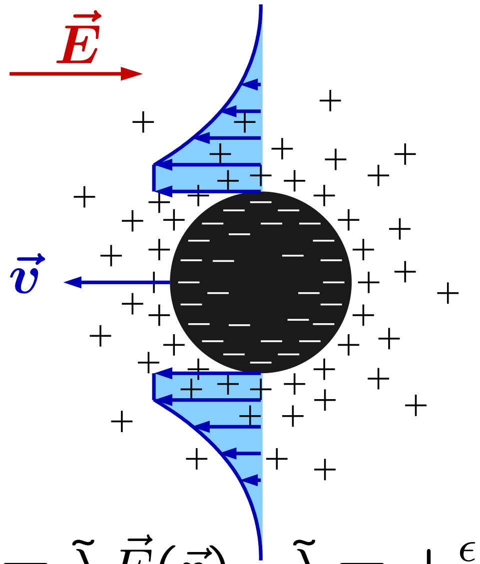

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$\Rightarrow$ general dynamics $$\boxed{m\,\ddot{\vec{r}}(t) = \vec{F}(\vec{r}(t)) - \eta\left[\dot{\vec{r}}(t) - \vec{v}_0(\vec{r}(t))\right]}$$

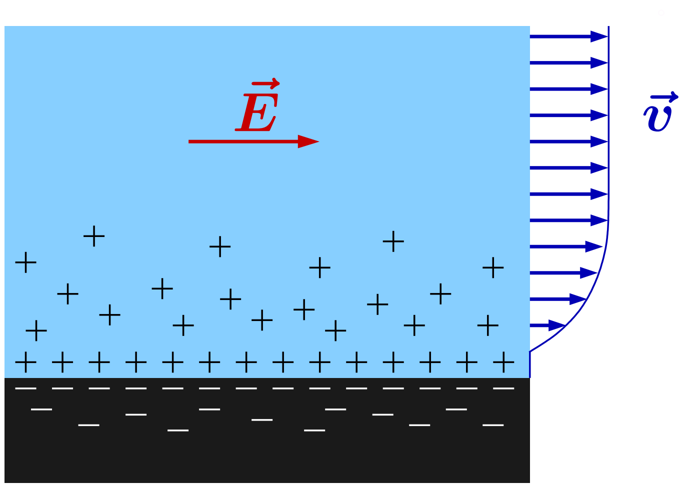
[Long, Viovy, Ajdari 1996]

m particle mass: negligible (overdamped)

η viscous friction coefficient

$\vec{F}(\vec{r})$ non-electric forces: hard walls $-\nabla V(\vec{r})$, thermal noise $\vec{\xi}(t)$

Electroosmosis



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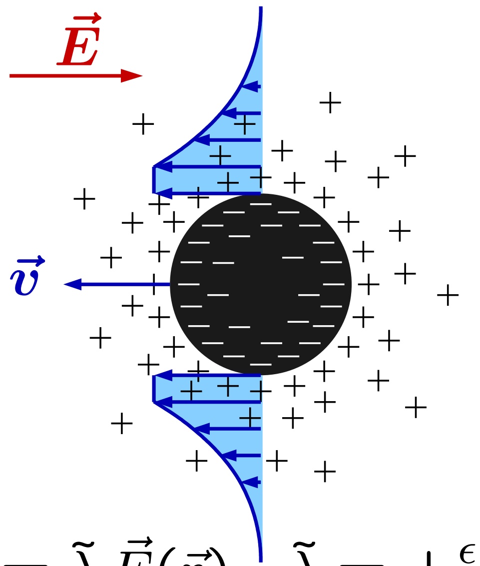

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$\Rightarrow$

$$\boxed{\eta\,\dot{\vec{r}}(t) = -\nabla V(\vec{r}(t)) + \vec{\xi}(t) + q_{\mathrm{eff}}\,\vec{E}(\vec{r}(t))}$$

$$q_{\mathrm{eff}} := \frac{\eta\,\epsilon\,(\tilde{\zeta} - \zeta)}{\nu}$$

Quantitative Theory

$$\eta\,\dot{\vec{r}}(t) = -\nabla V(\vec{r}(t)) + \vec{\xi}(t) + q_{\mathrm{eff}}\,\vec{E}(\vec{r}(t))\ , \qquad q_{\mathrm{eff}} := \eta\,\epsilon\,(\tilde{\zeta} - \zeta)/\nu$$

$$\langle \xi_i(t)\,\xi_j(s)\rangle = 2\eta k T\,\delta(t-s)\,\delta_{ij}\ , \quad T = 293\,\mathrm{K}$$

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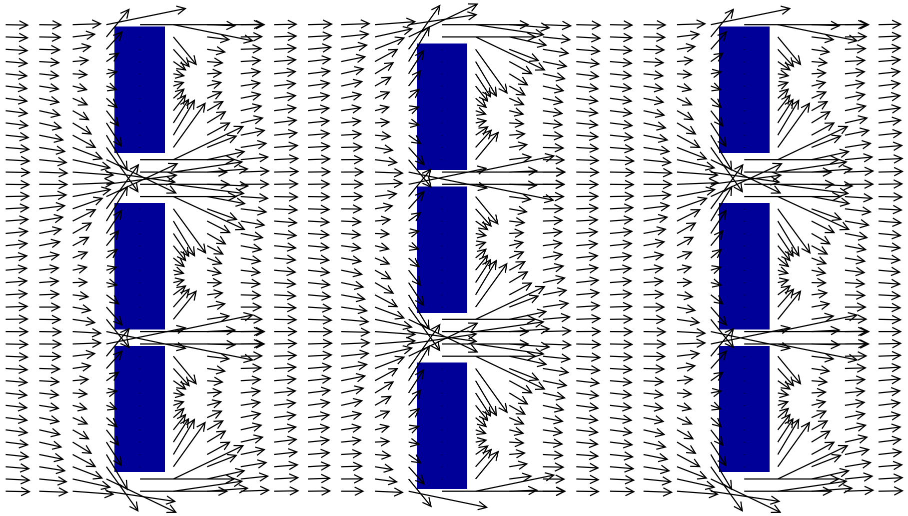
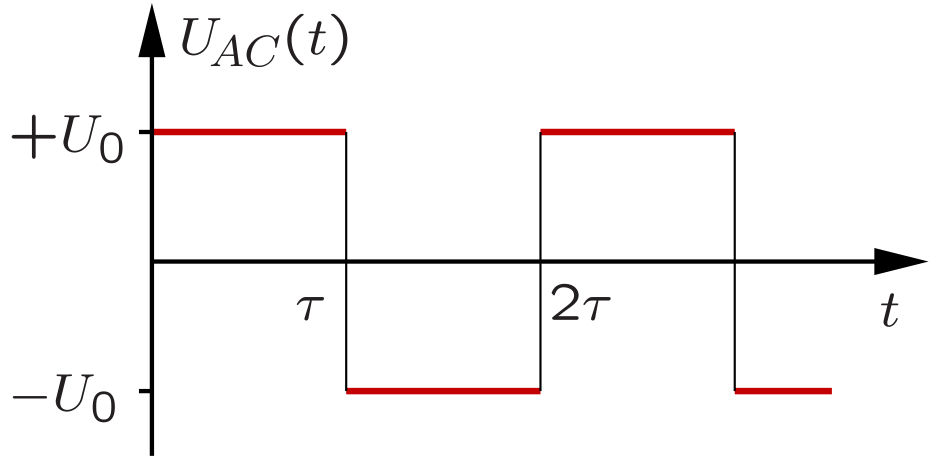
$\vec{E}_*(\vec{r})$





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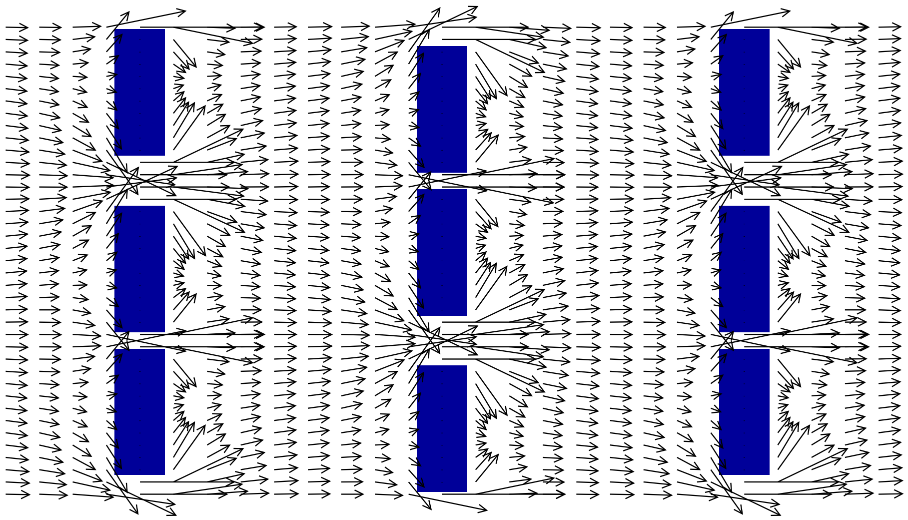
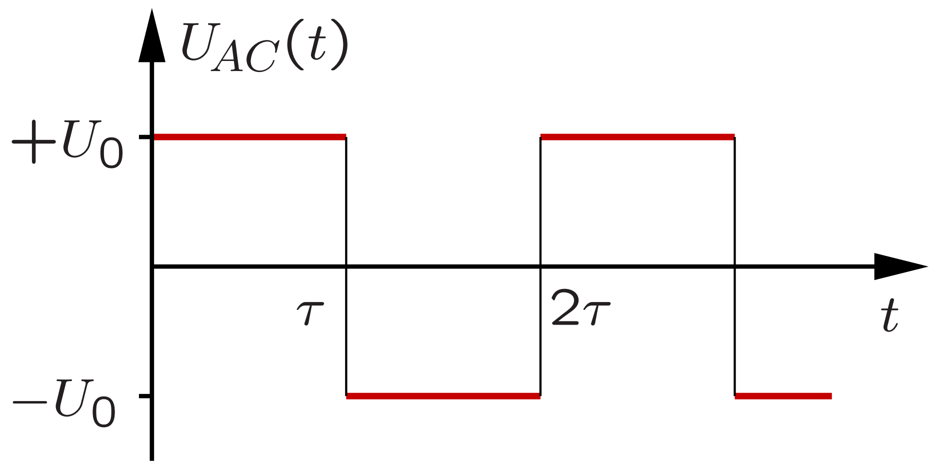
$\vec{E}_*(\vec{r})$





q_{eff}/U_* and η unknown

Theoretical determination practically impossible

Experimental determination by measuring
free mobility and **diffusion** ($D = kT/\eta$)

Kipp-Ratsche: Symmetrien

$$\boxed{\gamma\dot{x} = \xi(t) - V'(x) + F(t)} \quad V(x+L)=V(x),\ F(t+\tau)=F(t),\ \int_0^\tau F(t)dt=0$$

Kipp-Ratsche: Symmetrien

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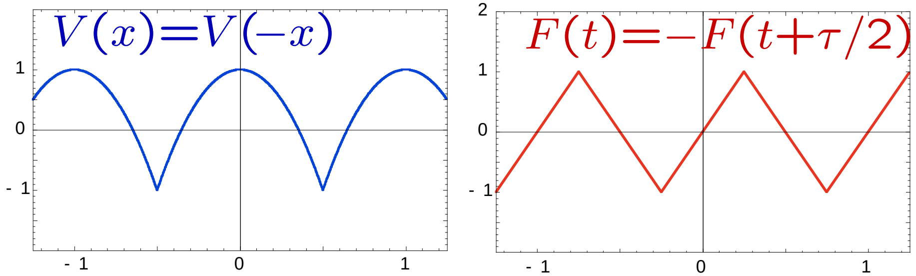
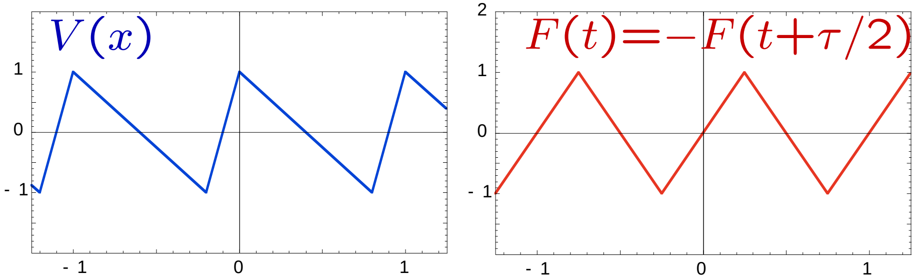
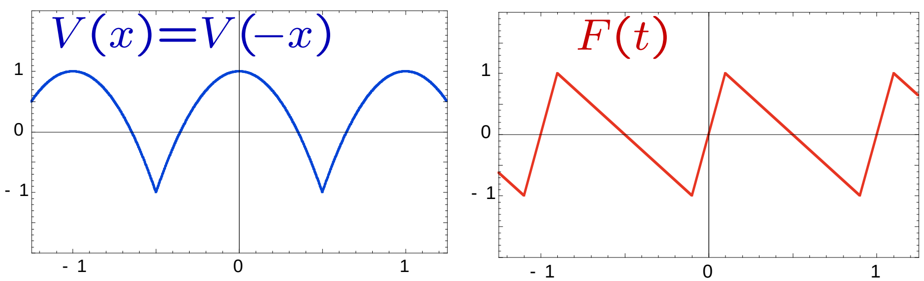


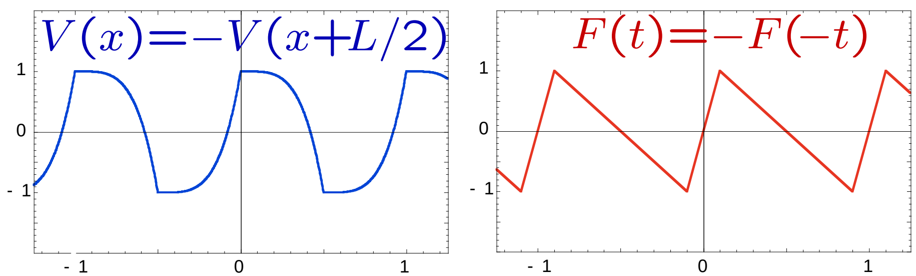
$\langle\dot{x}\rangle = 0$: **volle Symmetrie**



$\langle\dot{x}\rangle \neq 0$: **intrinsische Asymmetrie**



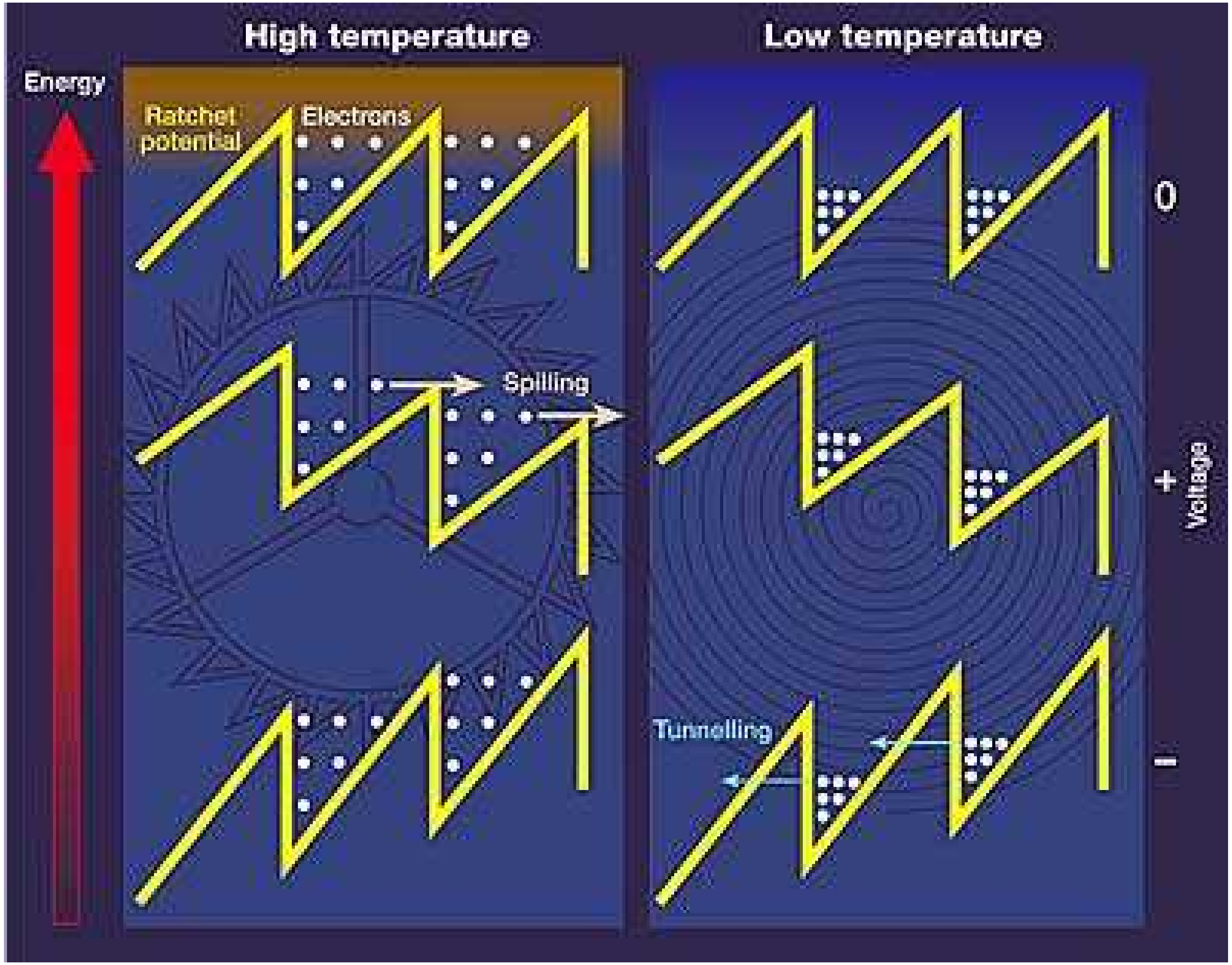
$\langle\dot{x}\rangle \neq 0$: **dynamische Asymmetrie**



$\langle\dot{x}\rangle = 0$: **Supersymmetrie**

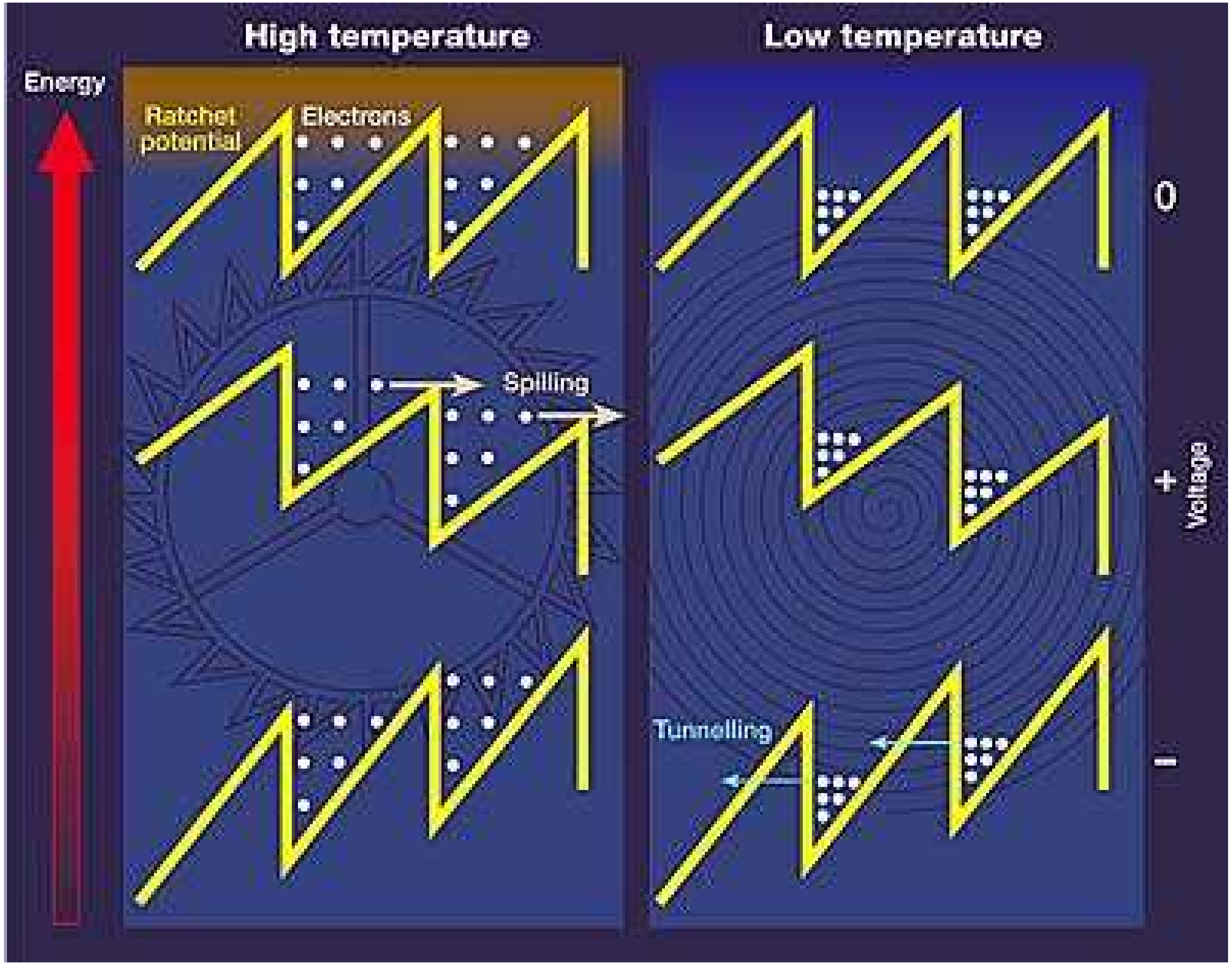
[P.R., PRL **86**, 4992 (2001)]

Quantum-Ratchet



[Figure: M. Brooks, New Scientist **2222**, 28 (2000)]

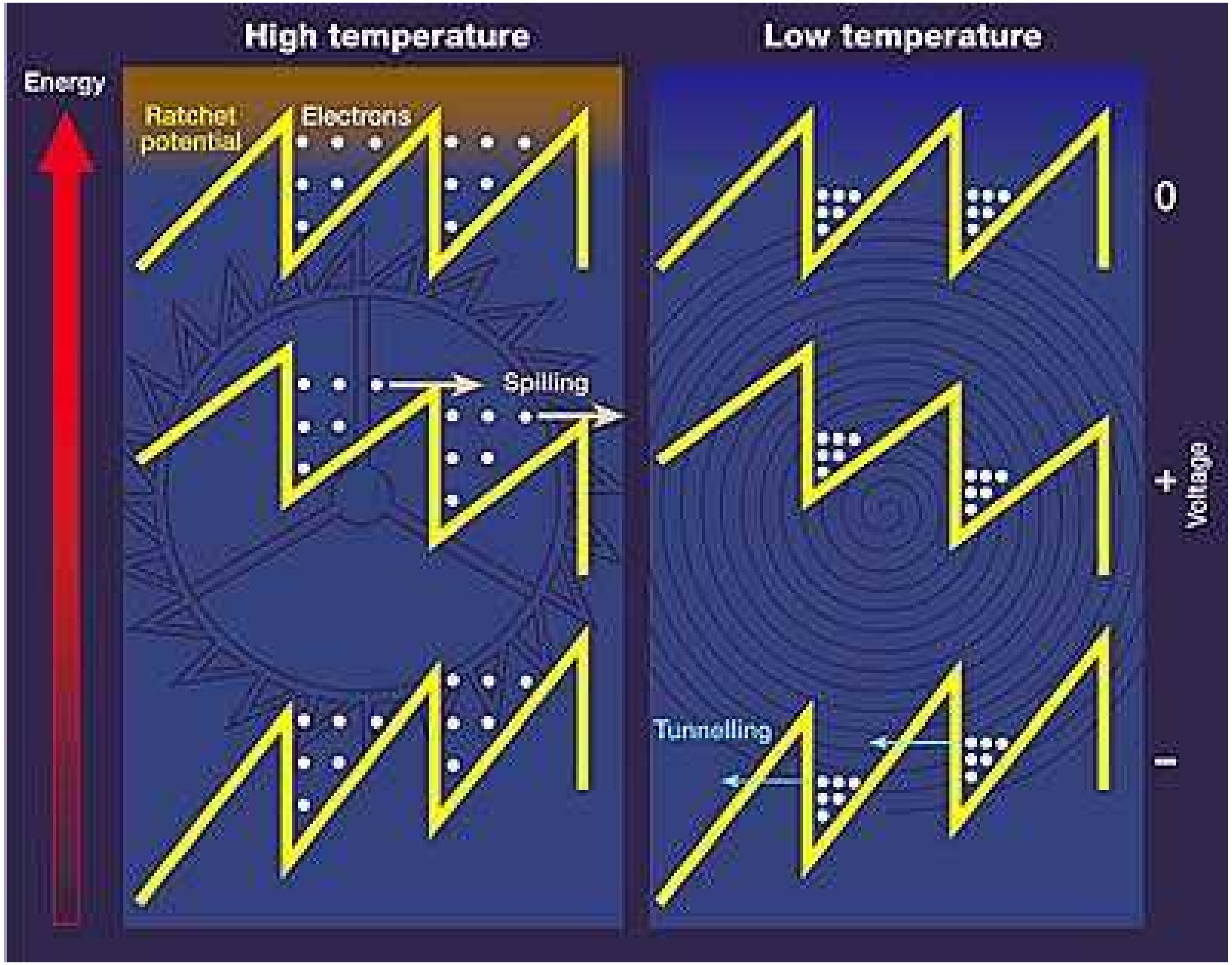
Quantum-Ratchet



[Figure: M. Brooks, New Scientist **2222**, 28 (2000)]

high $T \Rightarrow \langle \dot{x} \rangle > 0$

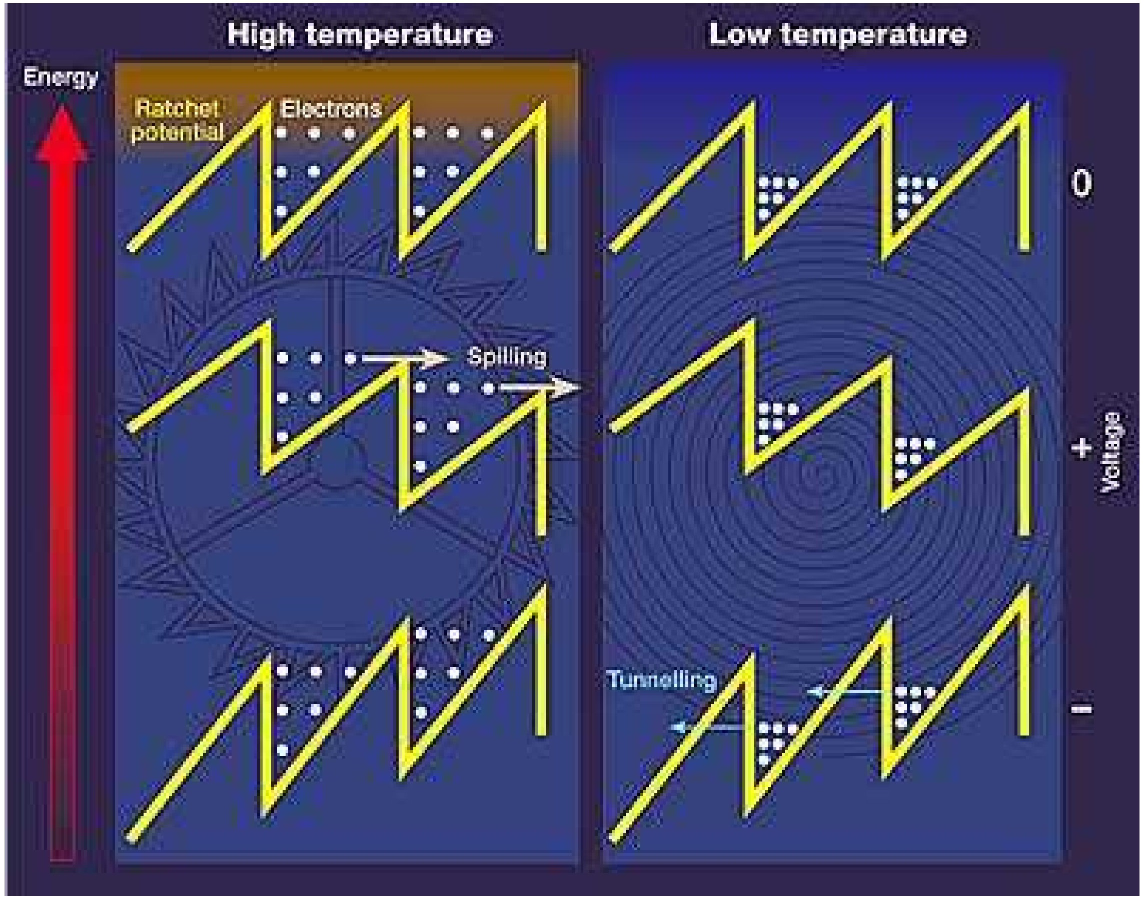
Quantum-Ratchet



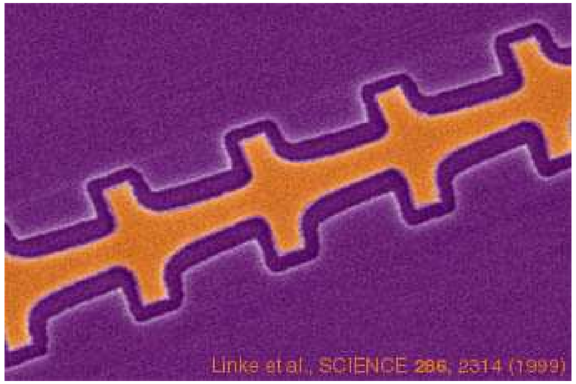
[Figure: M. Brooks, New Scientist **2222**, 28 (2000)]

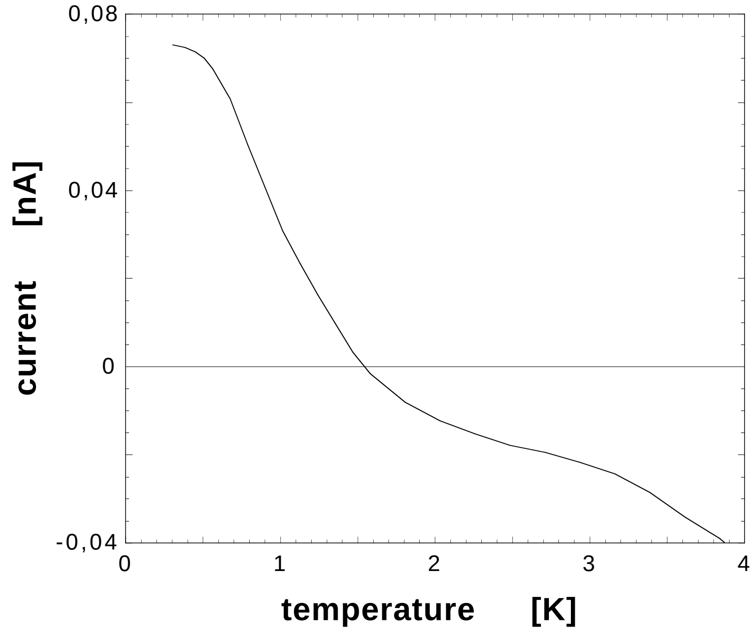
high $T \Rightarrow \langle \dot{x} \rangle > 0$ low $T \Rightarrow \langle \dot{x} \rangle < 0$

Quantum-Ratchet



[Figure: M. Brooks, New Scientist **2222**, 28 (2000)]





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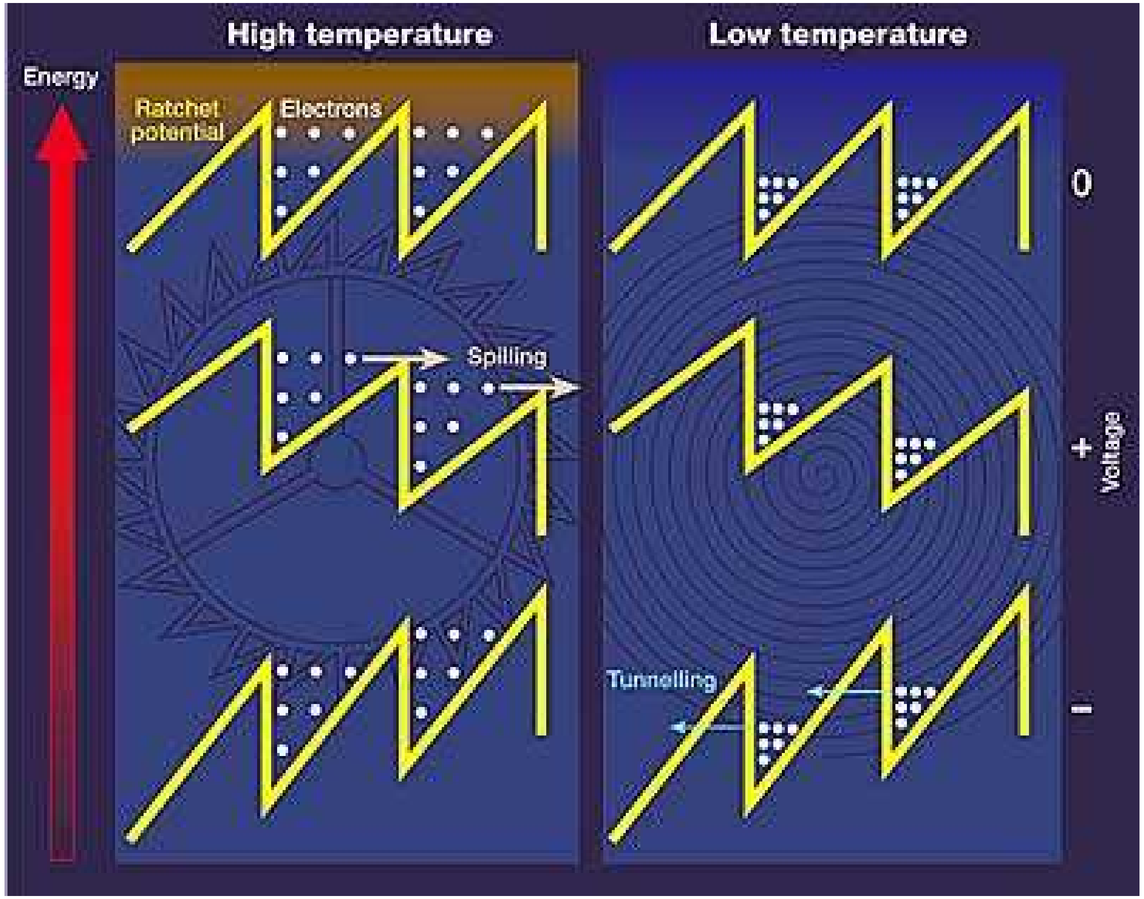
Theory: Reimann et al., Phys. Rev. Lett. **79**, 10 (1997)

Experiment: Linke et al., Science **286**, 2314 (1999)

1. Beispiel: Quanten-Ratsche



[Figur: M. Brooks, New Scientist **2222**, 28 (2000)]

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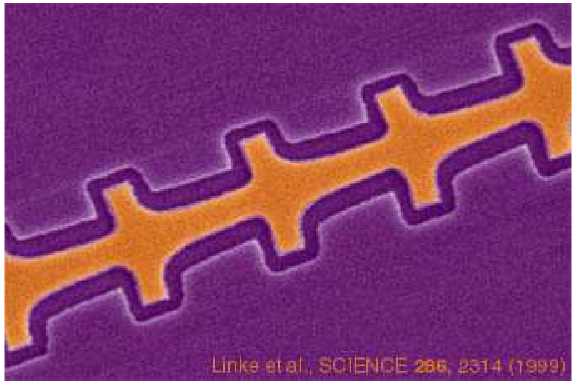
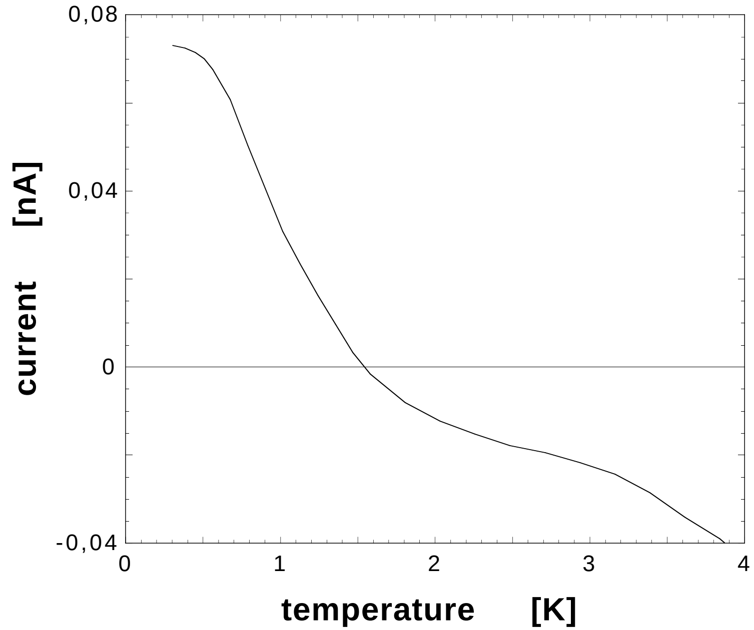


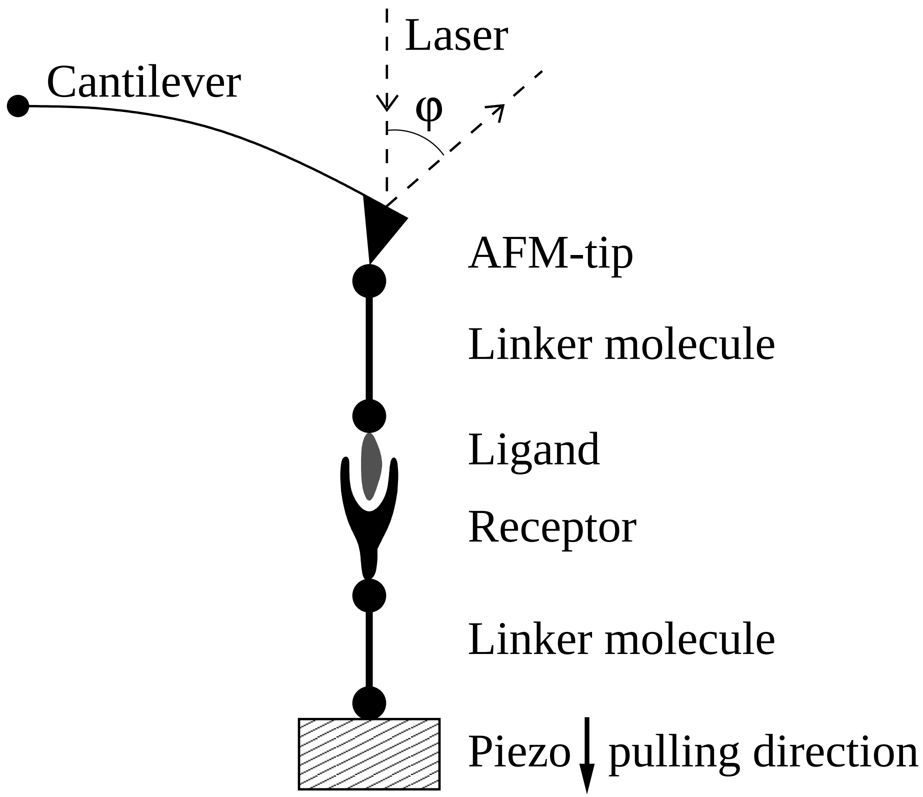


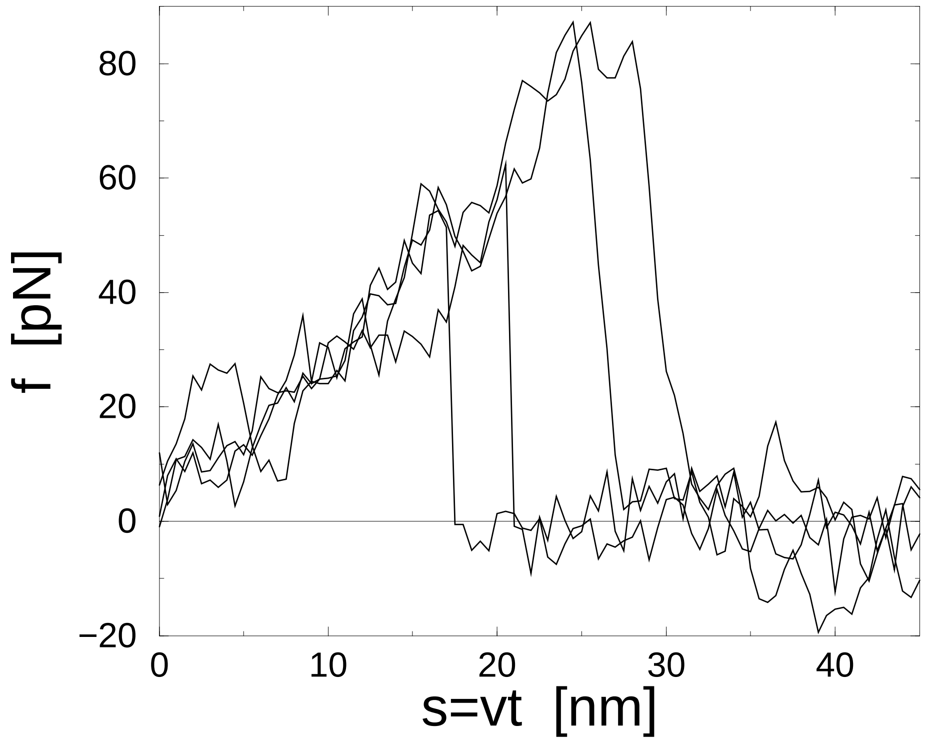
Theorie: Reimann et al., Phys. Rev. Lett. **79**, 10 (1997)
Experiment: Linke et al., Science **286**, 2314 (1999)

Einzelmolekül-Kraftspektroskopie

Experimente: R. Ros, D. Anselmetti (Bielefeld); R. Merkel (Jülich)

Theorie: M. Raible, P. Reimann





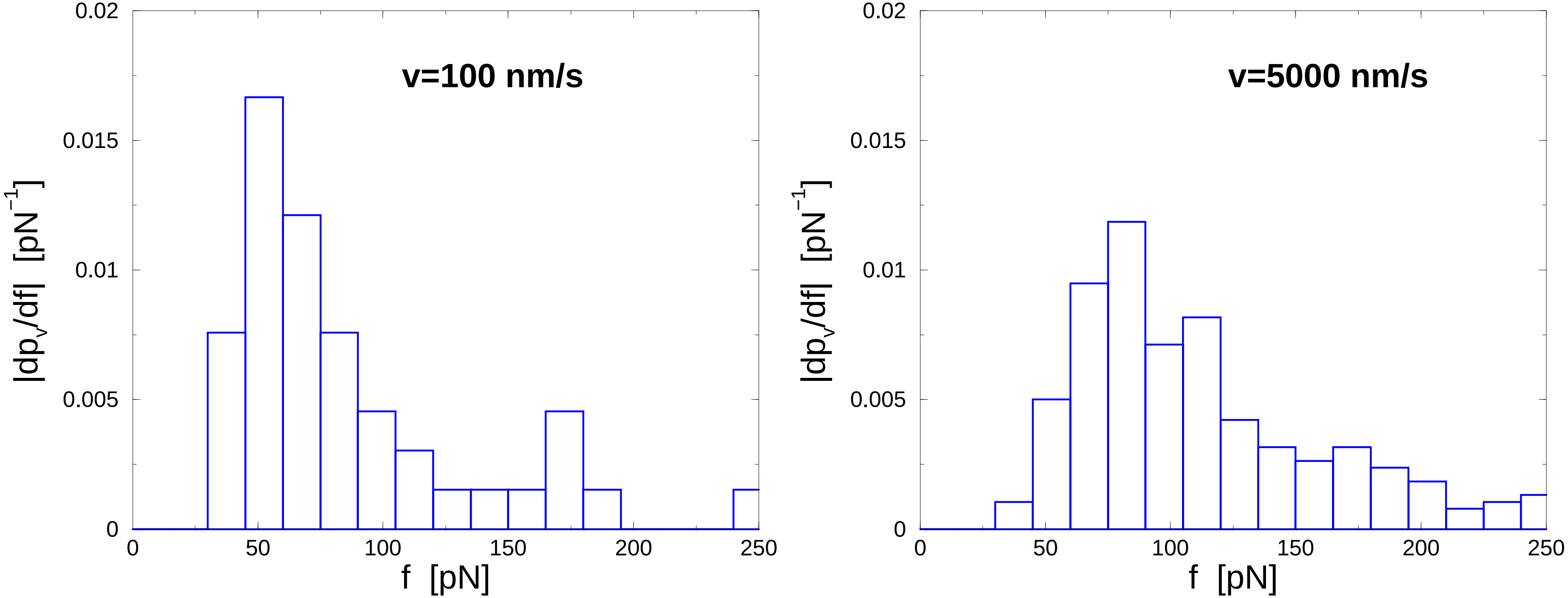
z.B. *expG*-Protein und expE-DNA

$f \simeq \kappa\, s\ , \quad \kappa \simeq 3$ pN/nm

Zentrale Grösse: **Verteilung der Abreisskräfte**

Verteilung der Abreisskräfte

(*expG*-Protein und expE-DNA)



Verteilung der Abreisskräfte ist abhängig von Ziehgeschwindigkeit v:

Interpretation ?

Standard-Theorie von Evans & Ritchie

[Biophys. J. **72**, 1541 (1997); ca. 400 mal zitiert]

$$\dot{p}_v(f(t)) = -r(f(t))\, p_v(f(t))$$

$p_v(f)$ Überlebensw'keit, $r(f)$ Dissoziationsrate, $f(t)$ Kraft

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Theoretische Voraussage:

[Raible, Evstigneev, Reimann]

$\Rightarrow$ $-v \ln p_v(f)$ unabhängig von v

[Phys. Rev. E **68**, 045103(R) (2003)]

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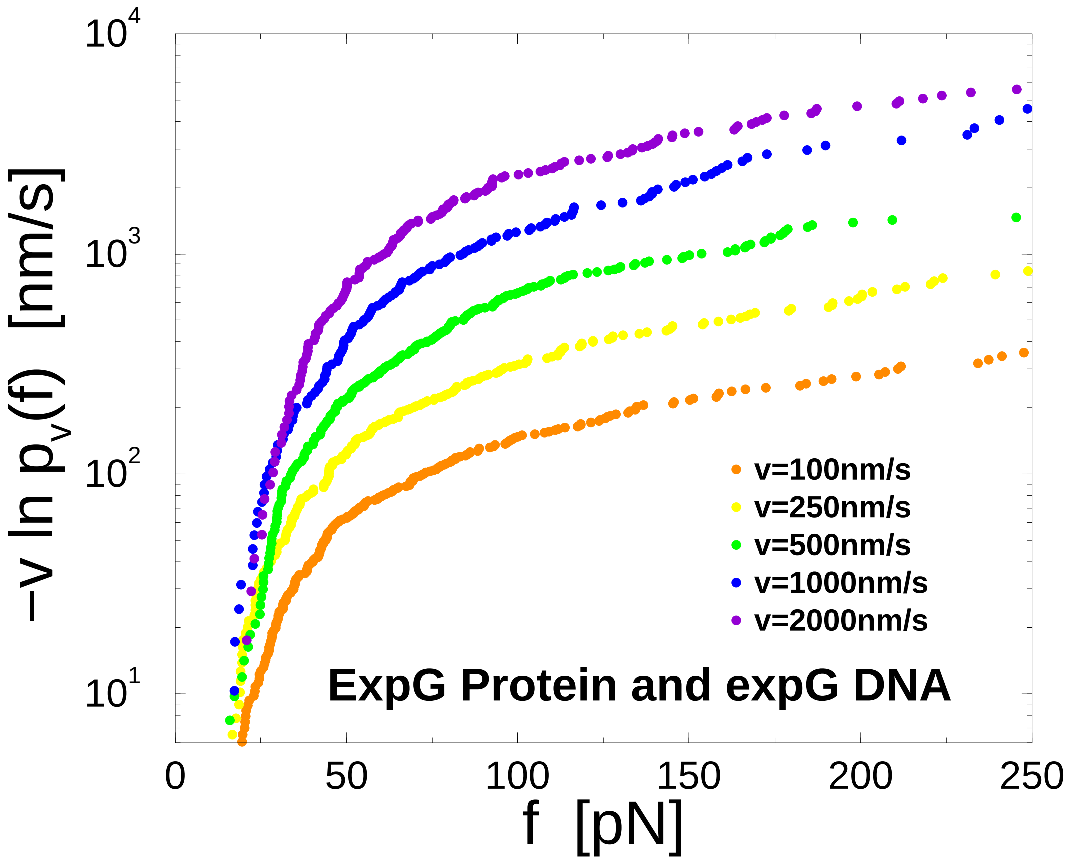
[Phys. Rev. E **68**, 045103(R) (2003)]

Vergleich mit Messdaten:

[von Ros & Anselmetti sowie Merkel]

$\Rightarrow$ **Standard-Theorie falsch !**

[J. Biotechnology **112**, 13 (2004)]



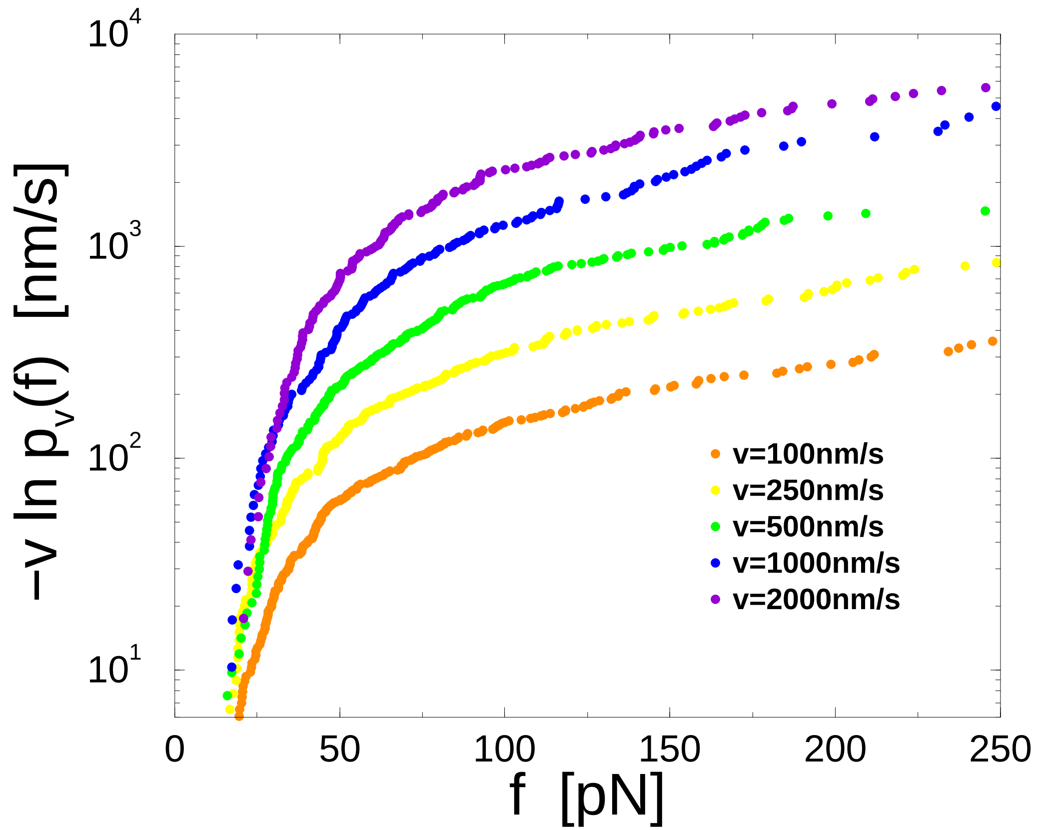
Neue Theorie

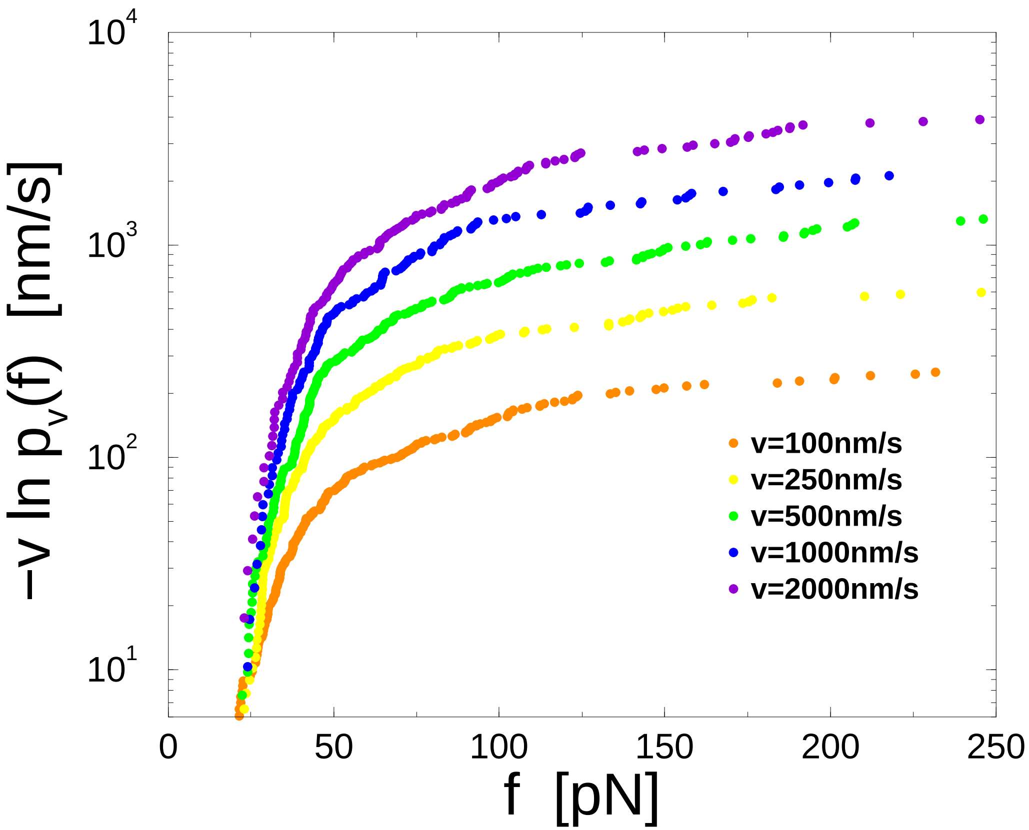
[Raible & Reimann, submitted]

Heterogenität der Dissoziationsrate

$r(f) \simeq r_o\, e^{\alpha f}$, α **Zufallsvariable**

$\langle\alpha\rangle \simeq 0.13$ pN^{-1}, $\sigma \simeq 0.07$ pN^{-1}, $r_o \simeq 0.0034$ s^{-1}





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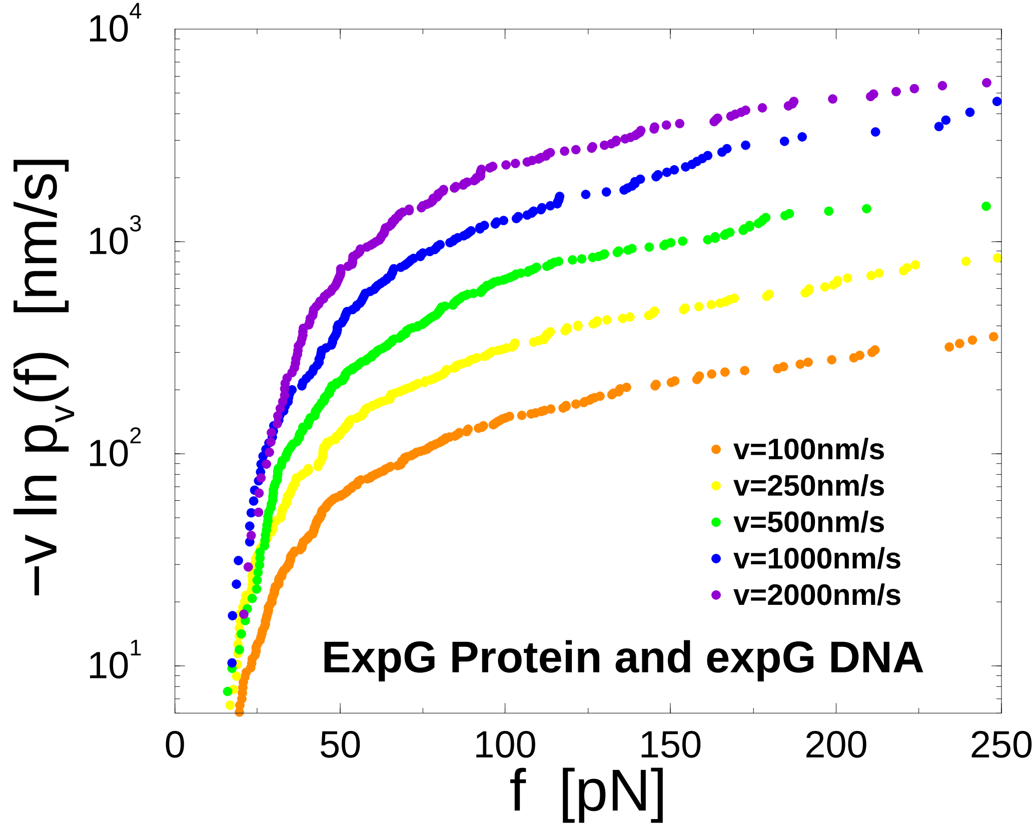
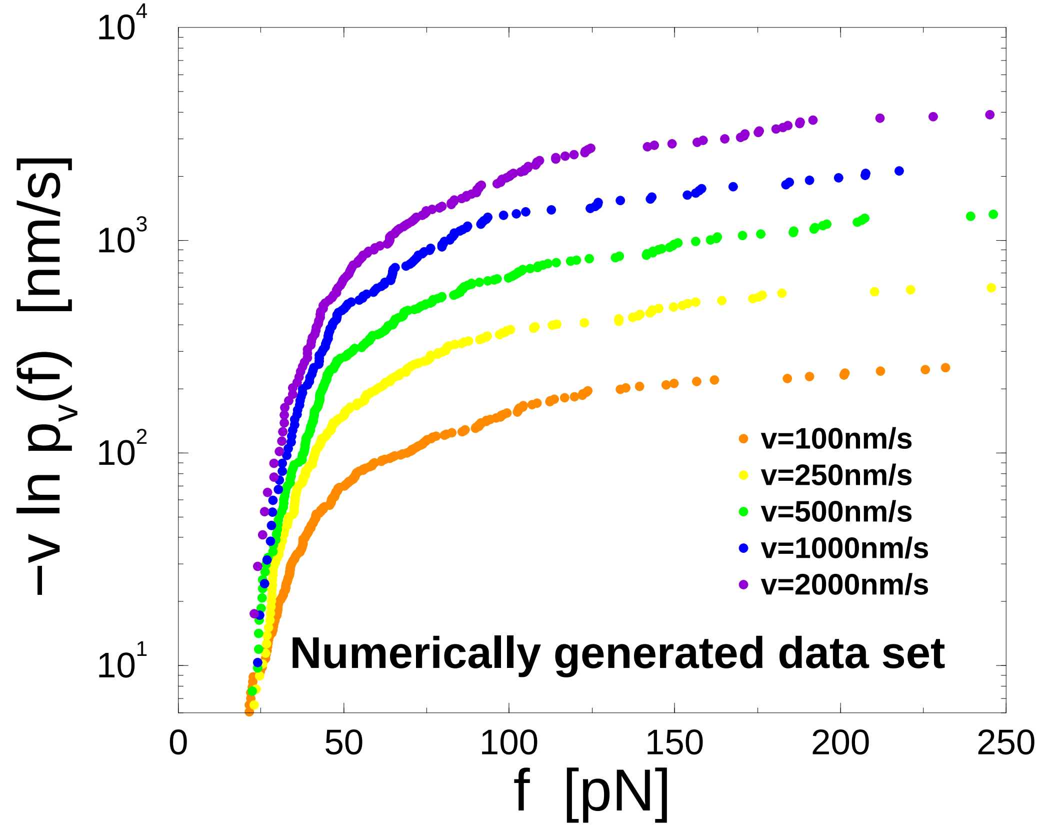


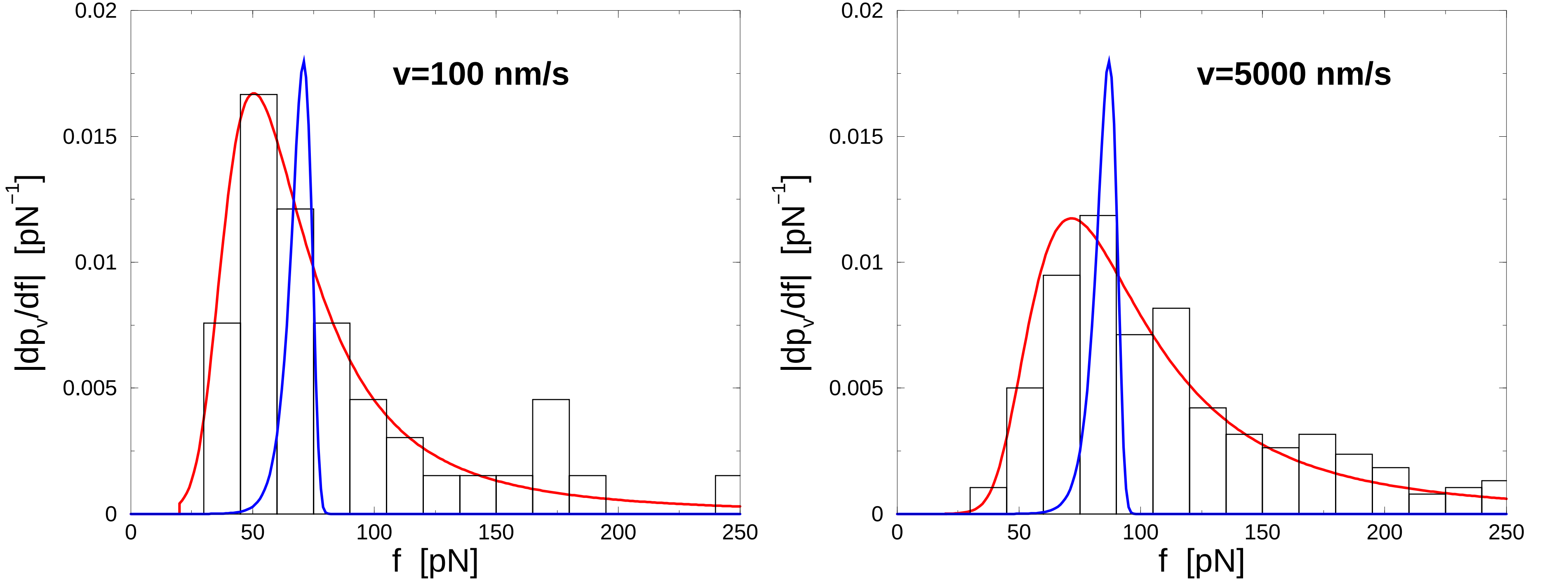


Verteilung der Abreisskräfte

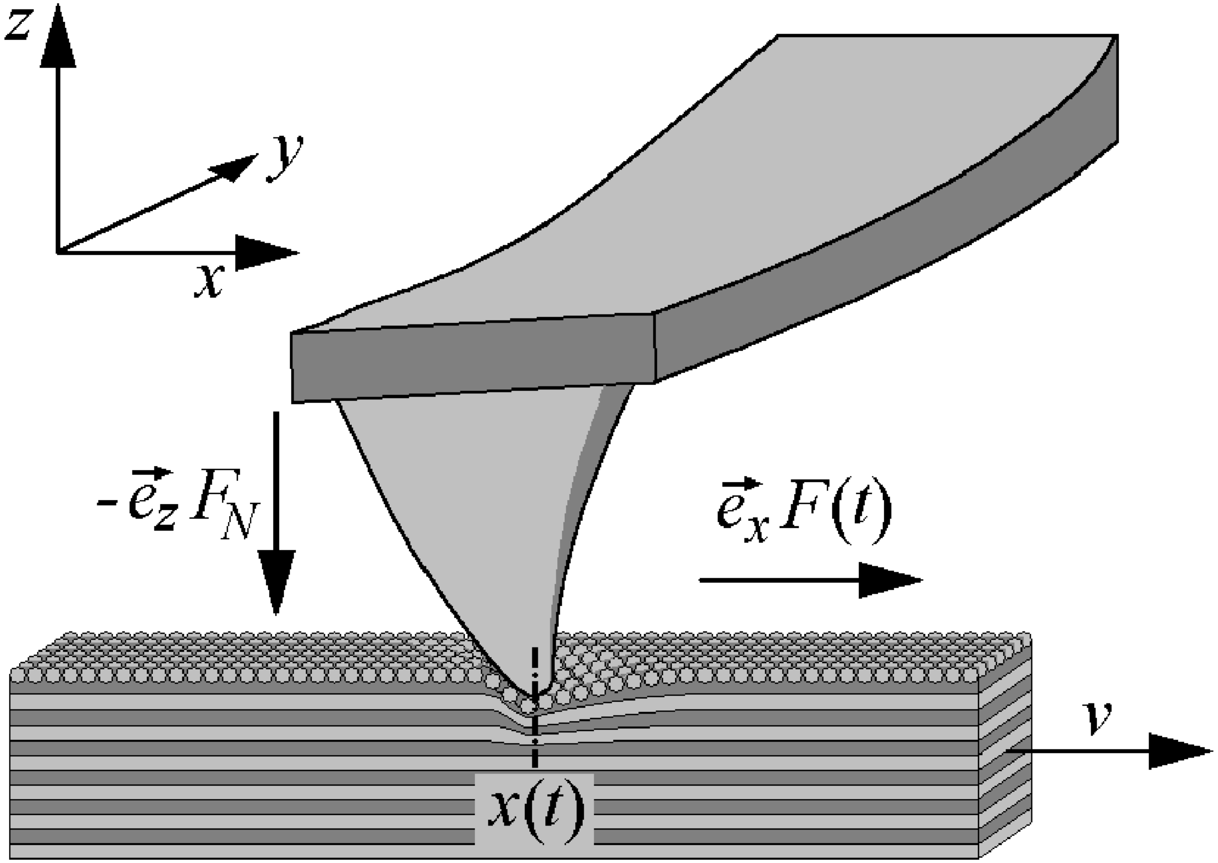
Experimentelle Messdaten: schwarze Histogramme

Standard-Theorie [Evans & Ritchie]: blaue Kurven

Neue Theorie [Raible & Reimann]: rote Kurven



Atomic Friction



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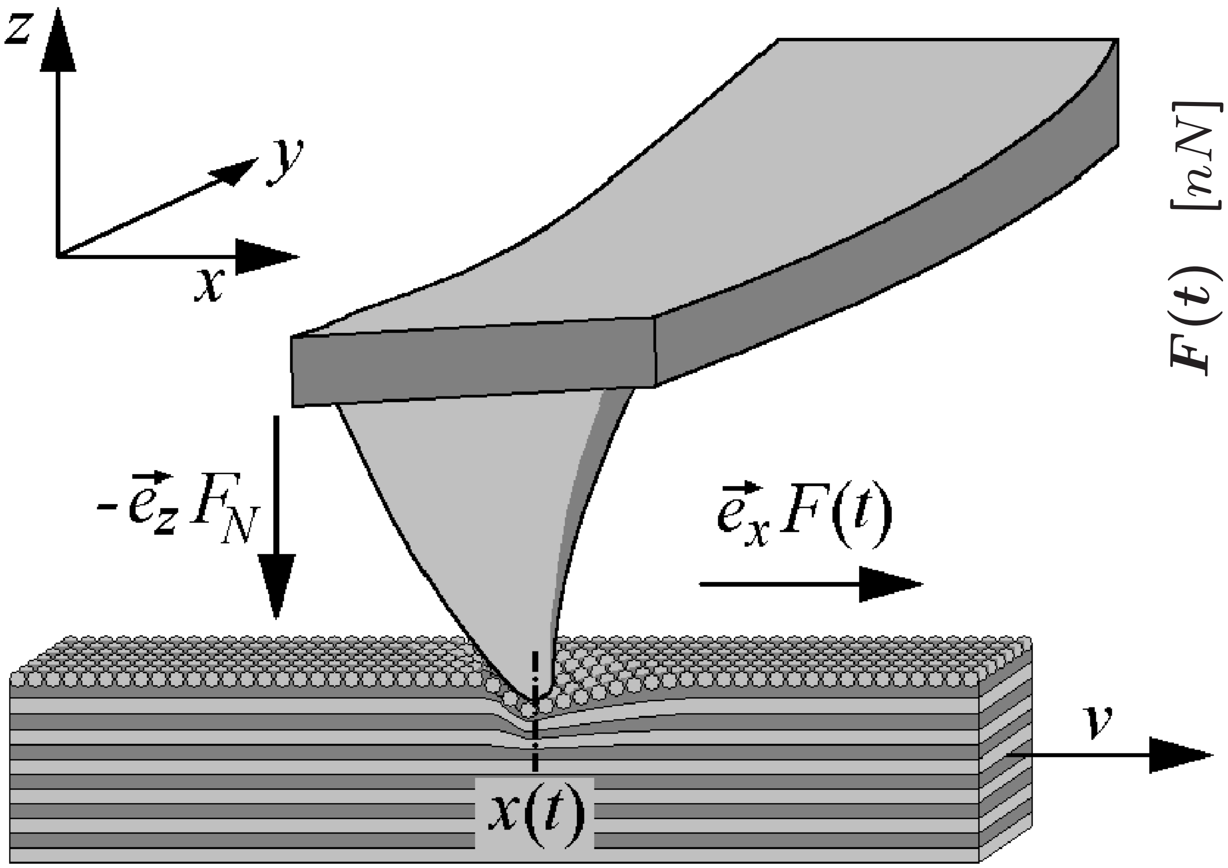
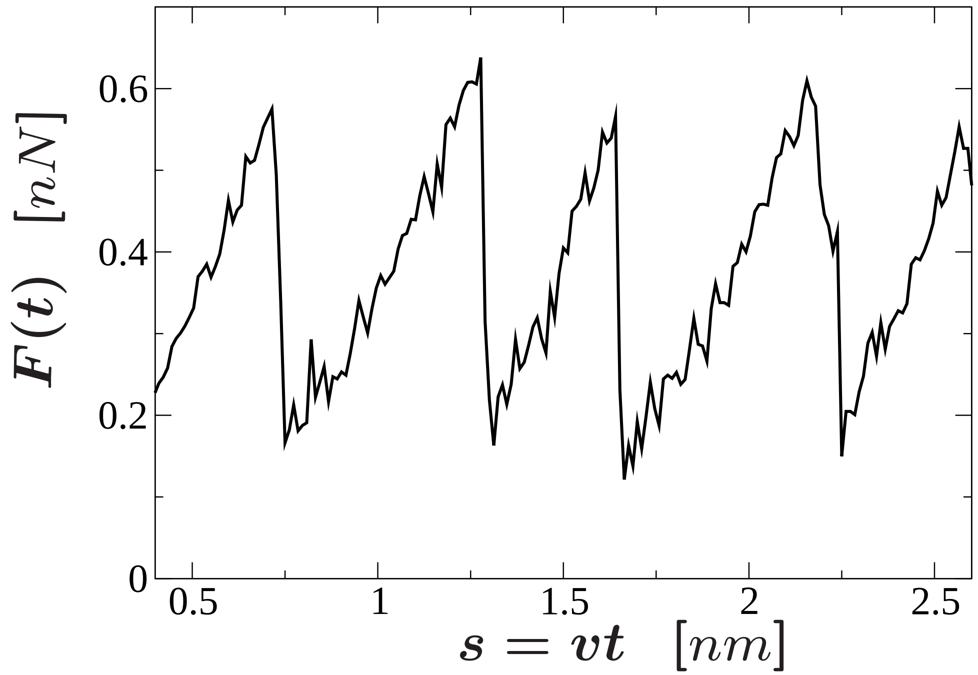
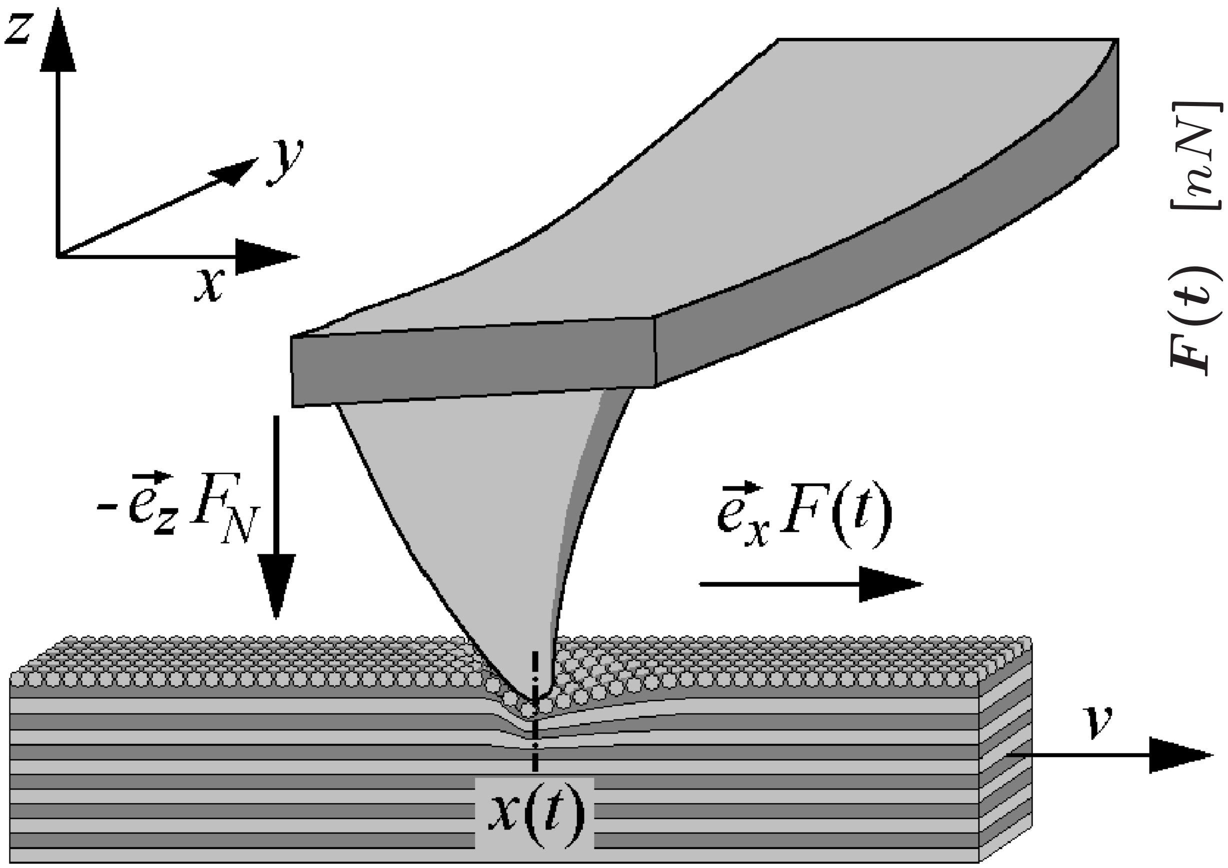
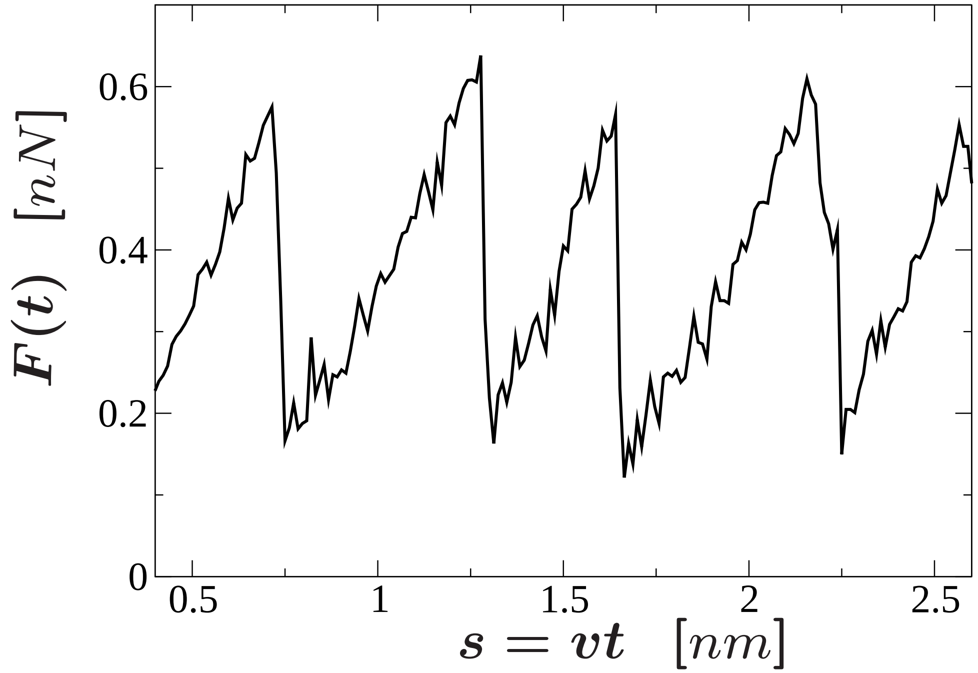




- stick-slip motion
- thermally activated

Atomic Friction





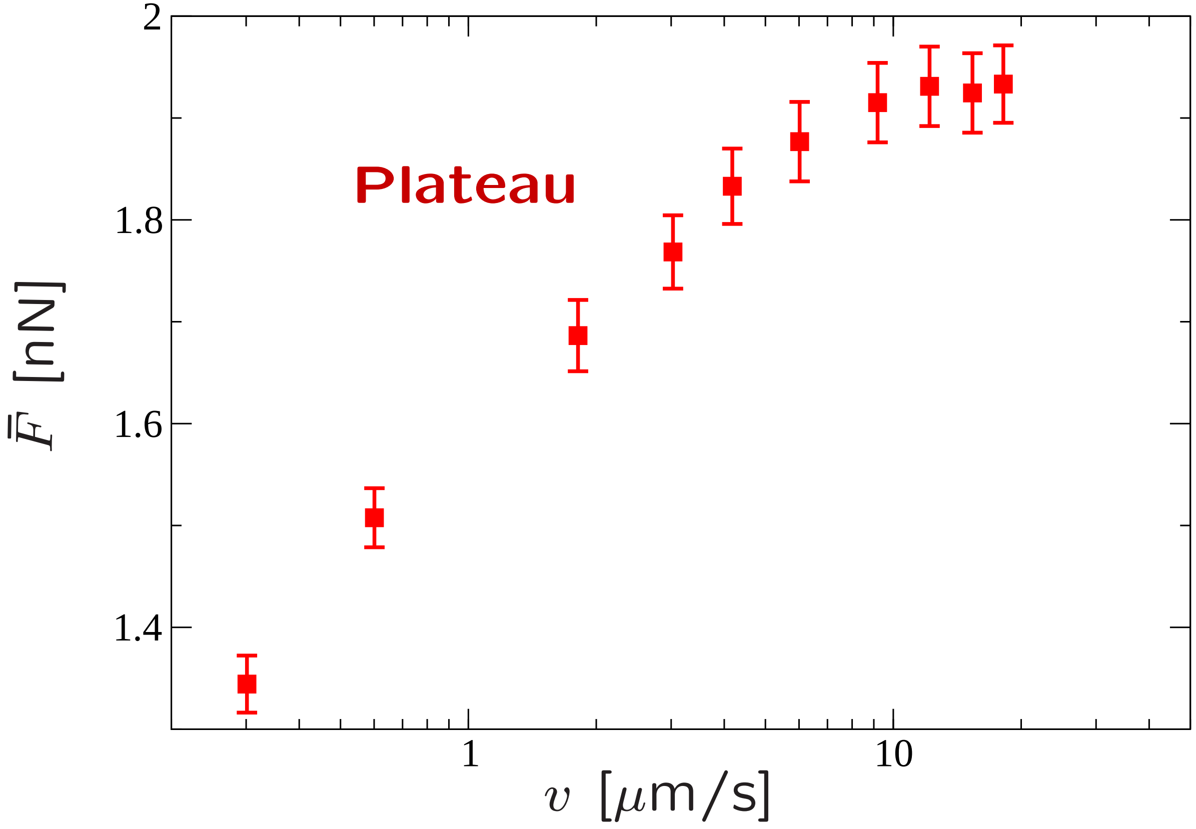
- stick-slip motion
- thermally activated

Quantity of main interest: $\bar{F} := \lim_{t\to\infty} \frac{1}{t} \int_0^t dt'\, F(t')$

Velocity Dependence of Atomic Friction

Experiment: Riedo et al., PRL 91, 084502 (2003)

(mica surface, $L = 0.52$ nm, $T = 293$ K, $F_N = 12$ nN)



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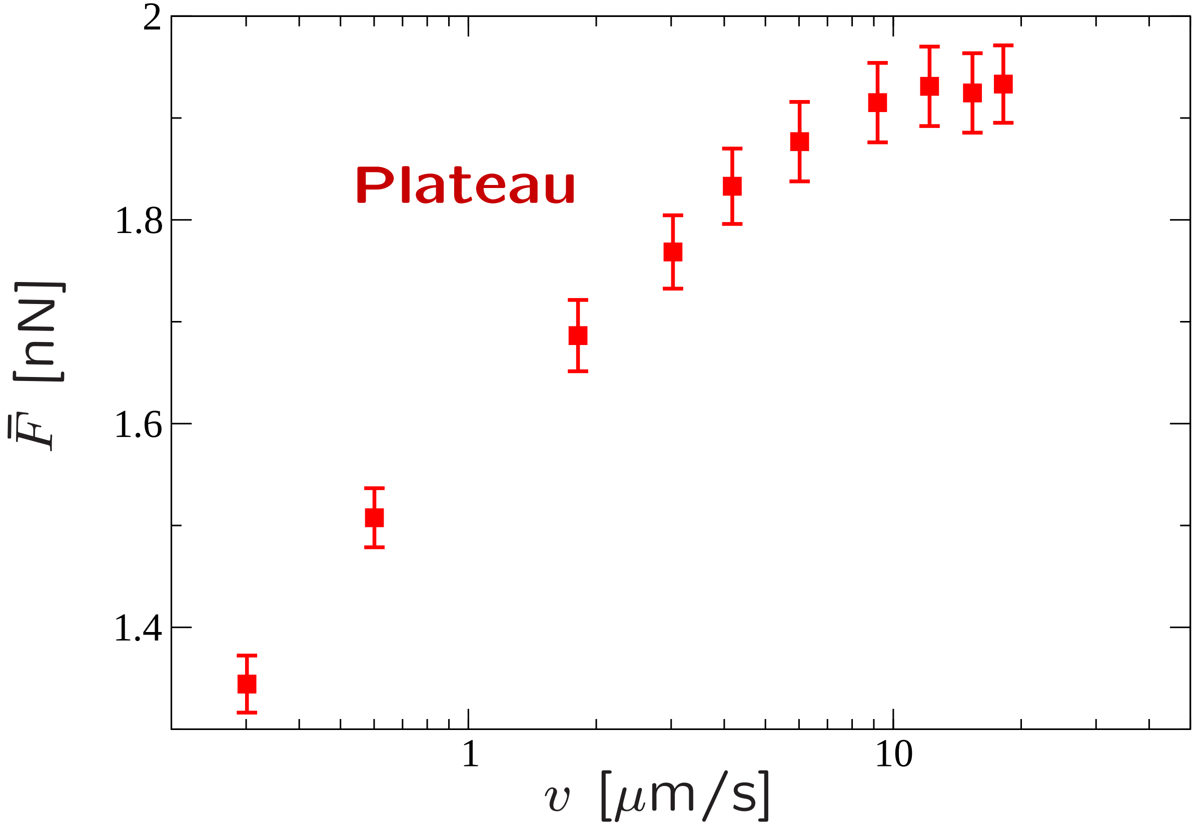

Theory: P. R. & Evstigneev, PRL 93, 230802 (2004)

$$v(f) \simeq \frac{L\,k\,T\,[1-e^{-Lf/kT}]}{\gamma \int_0^L dx \int_x^{x+L} dy \;\; e^{[U(x)-U(y)+(x-y)f]/kT}} \quad , \qquad \bar{F}(f) \simeq f - \gamma v(f)$$

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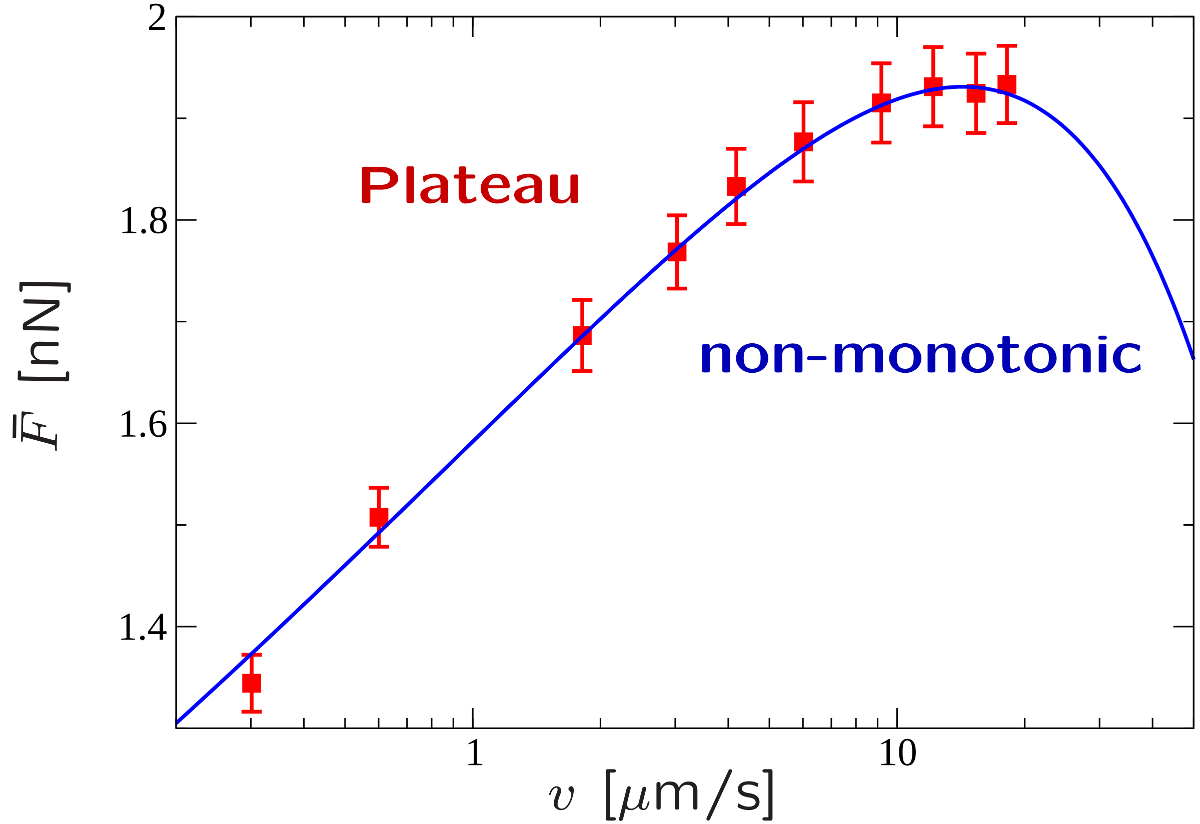

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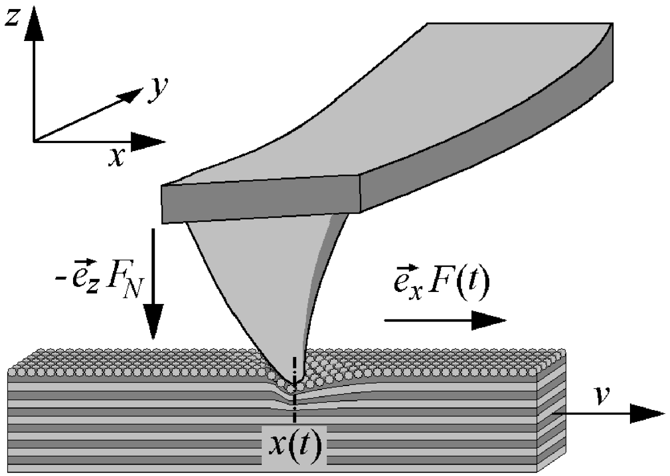
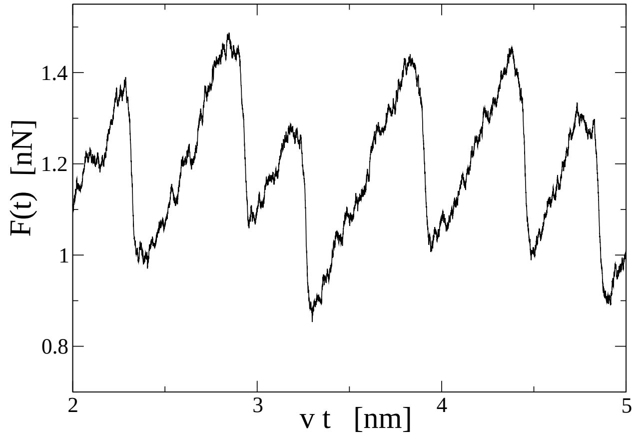
Physical Mechanism



$v = 30\,\mu m/s$



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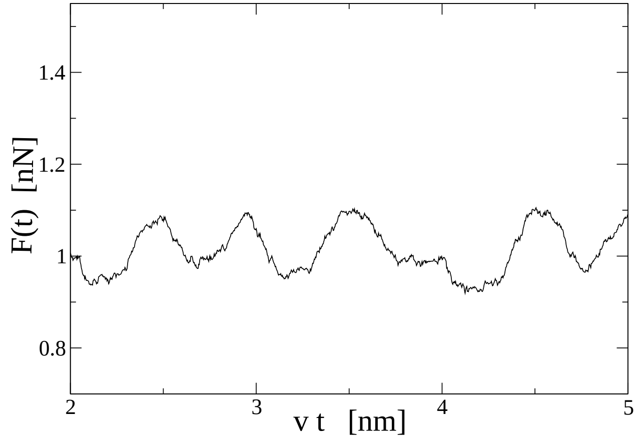

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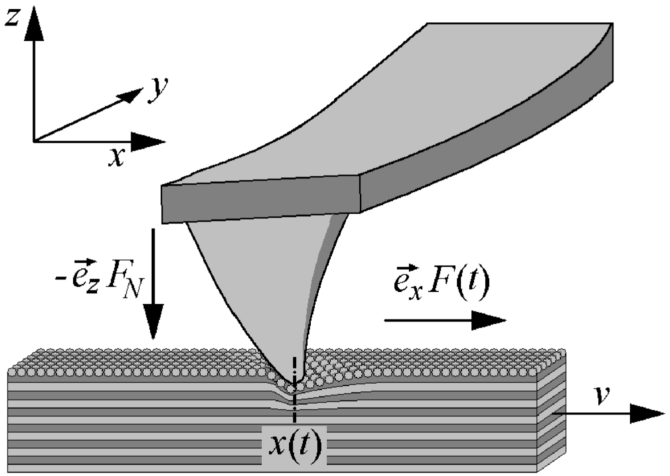

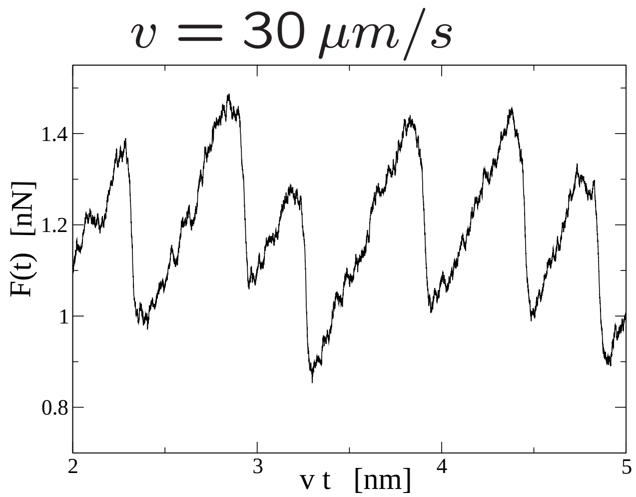

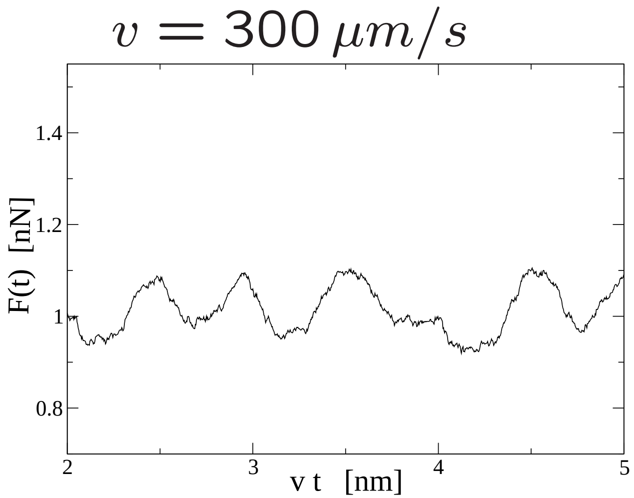

- stick-slip-amplitude $\widehat{=}$ dissipation

  $\Rightarrow \bar{F}(v)$ <u>decreasing</u>

- slips: thermally activated transitions

  $\Rightarrow \bar{F}(v)$ <u>increasing</u>

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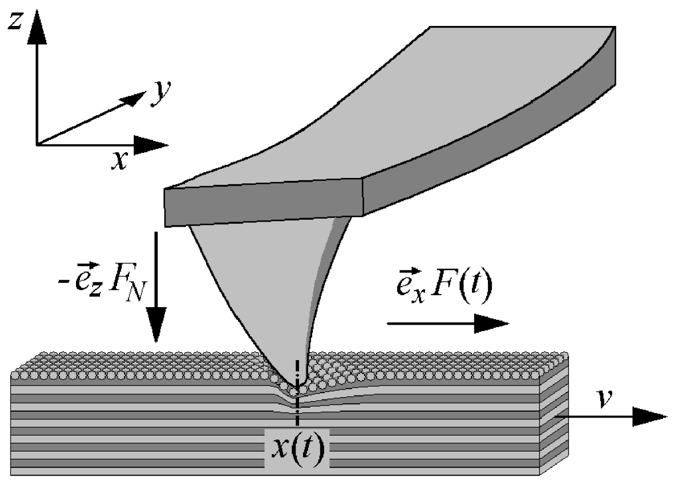

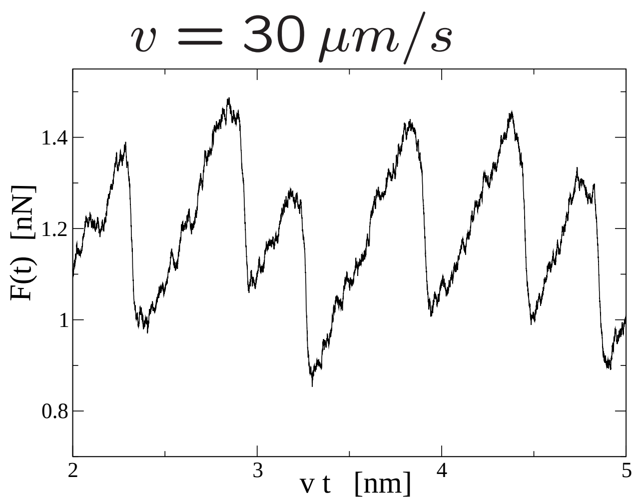

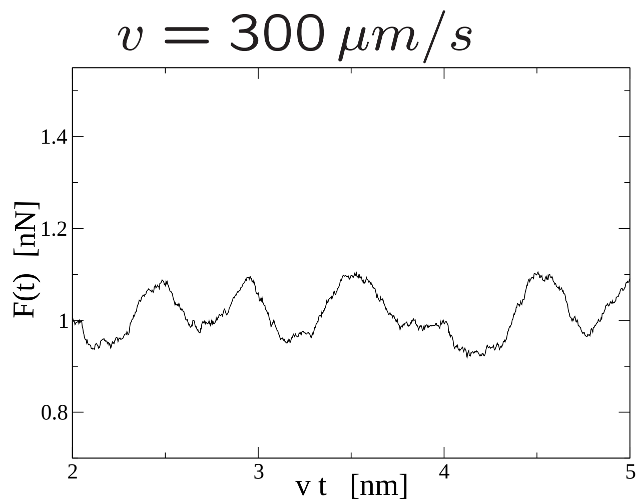

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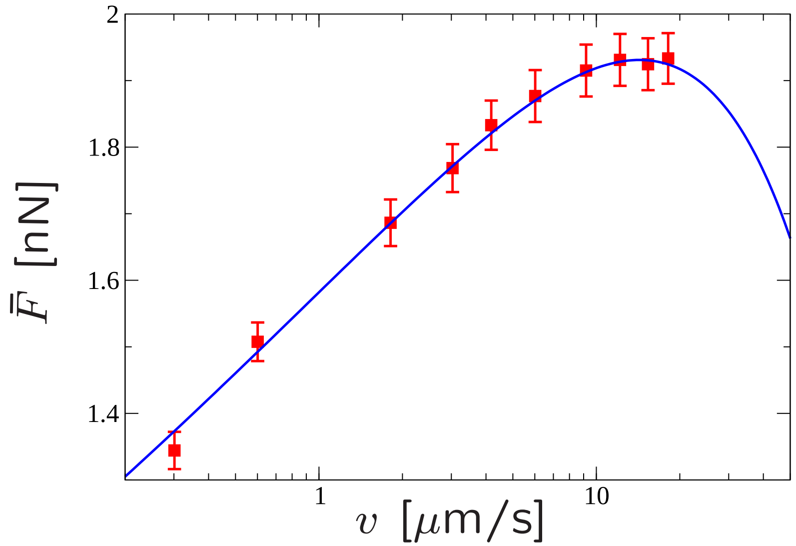
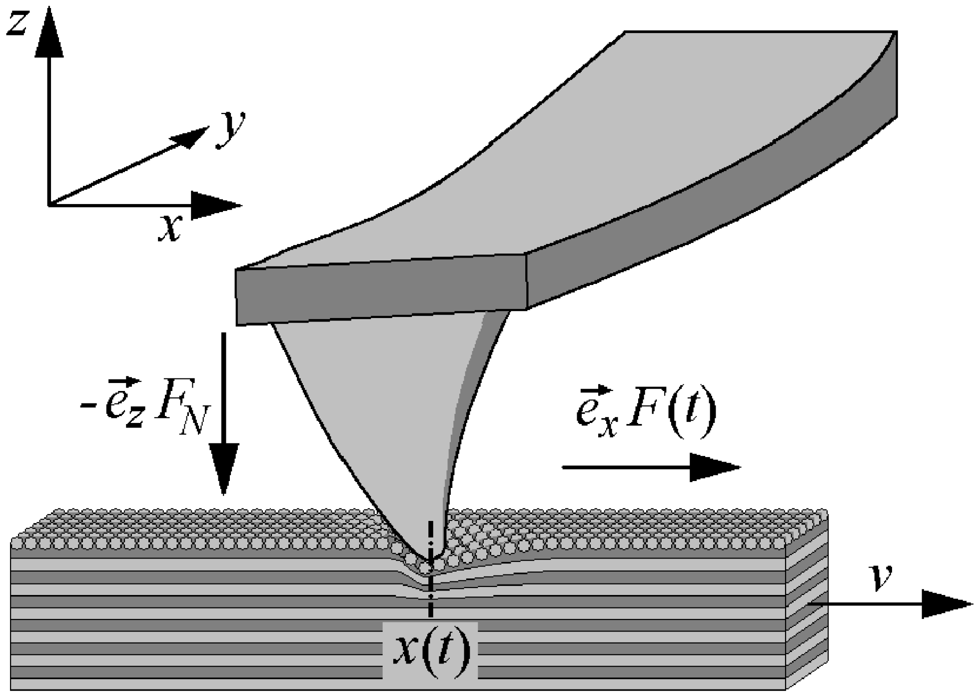
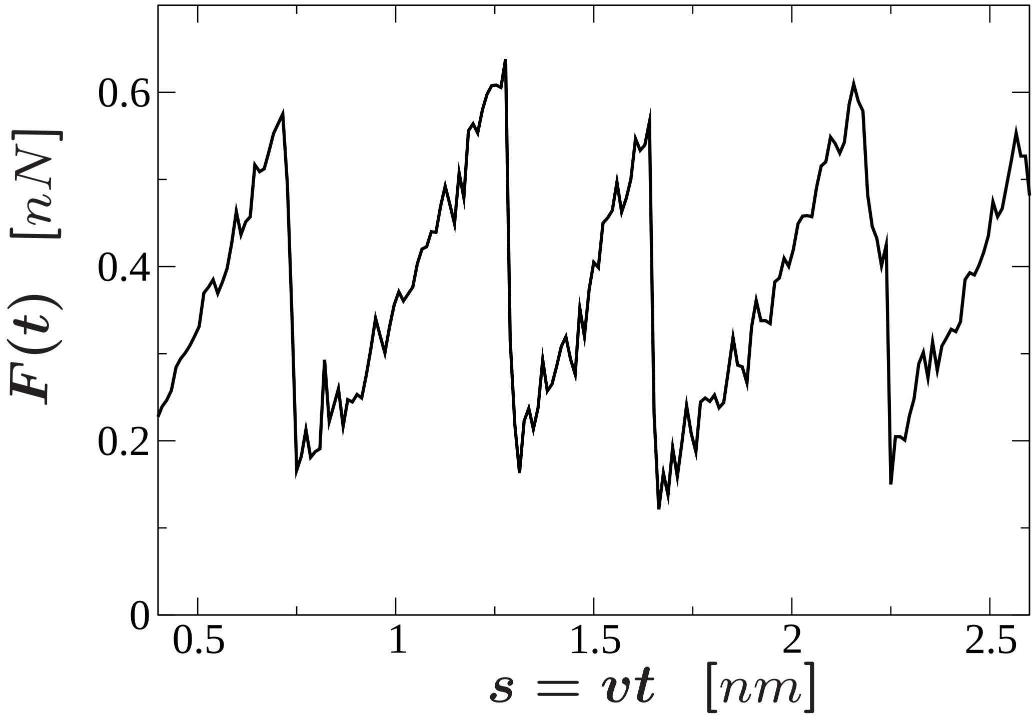
together **$\bar{F}(v)$ non-monotonic**



Slip Statistics and Rate Theory





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$F(t)$ instantaneous force, $r(F)$ “slip rate”, $p_v(F)$ “stick probability”

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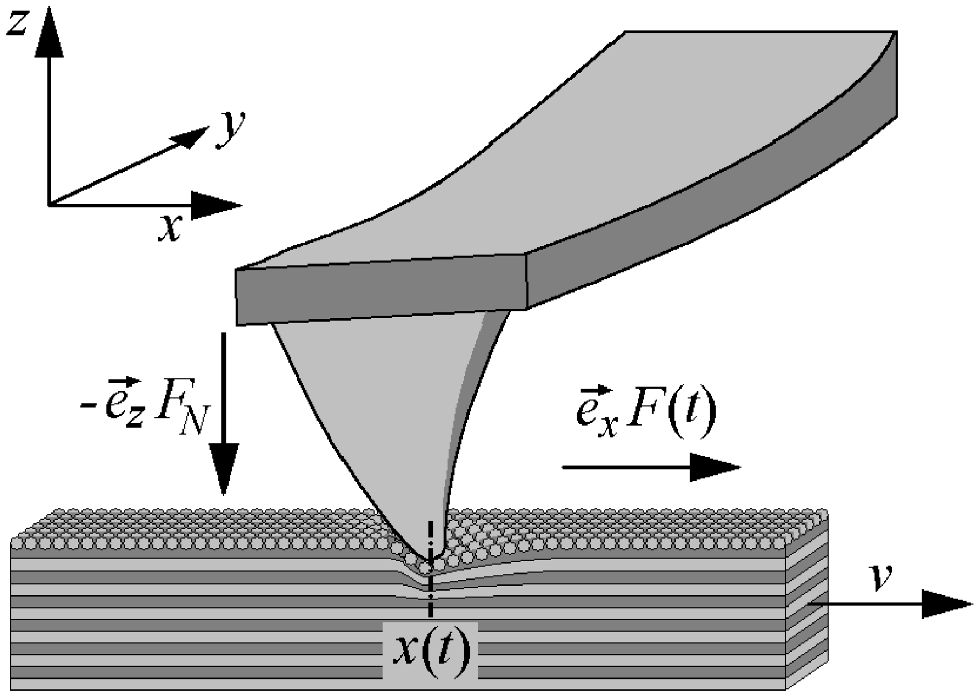

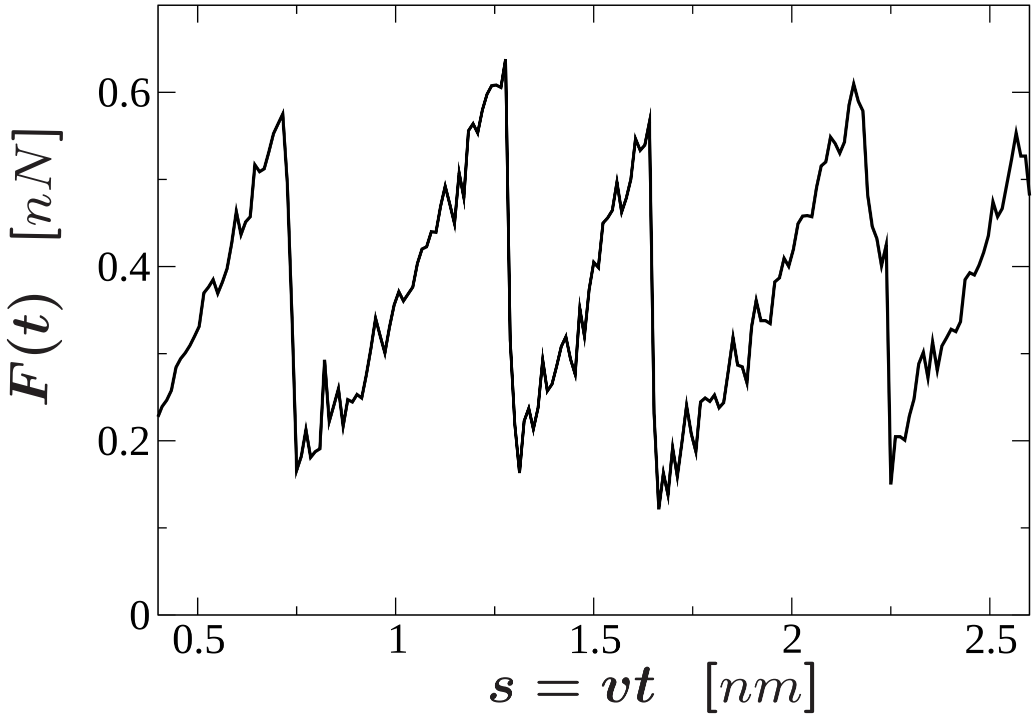

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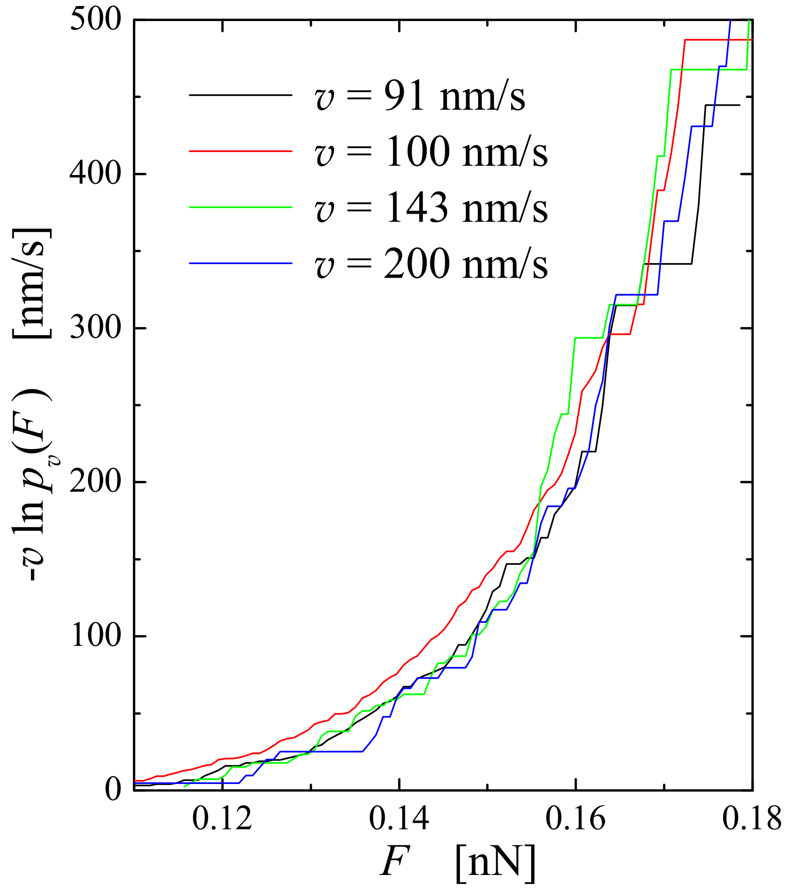
$\Rightarrow$ $-v \ln p_v(F)$ independent of v

Slip Statistics and Rate Theory

[Evstigneev, Schirmeisen, Jansen, Fuchs, P. R., PRL **97**, 240601 (2006)]



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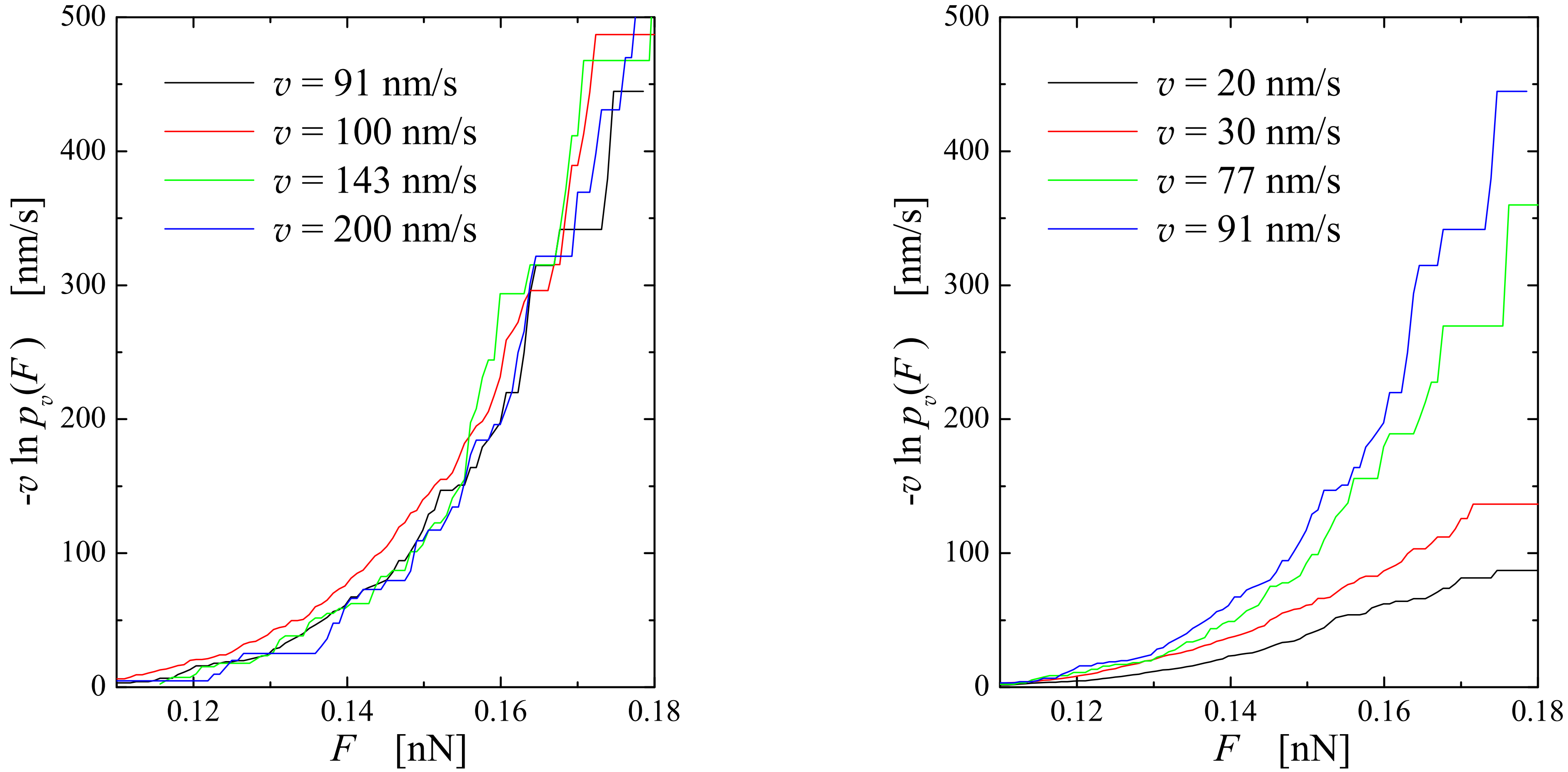

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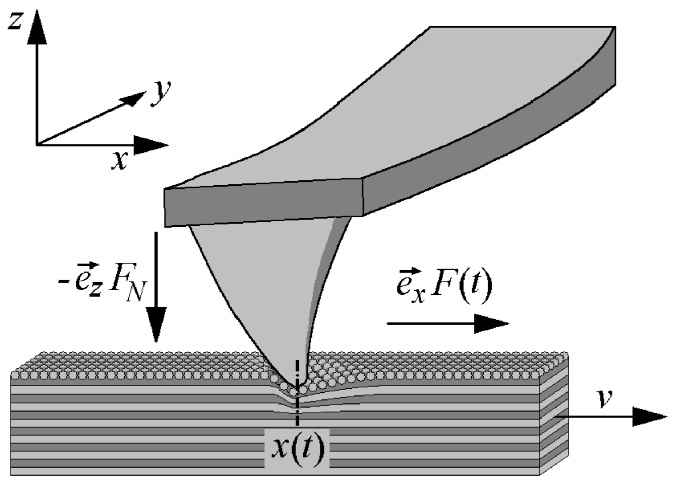

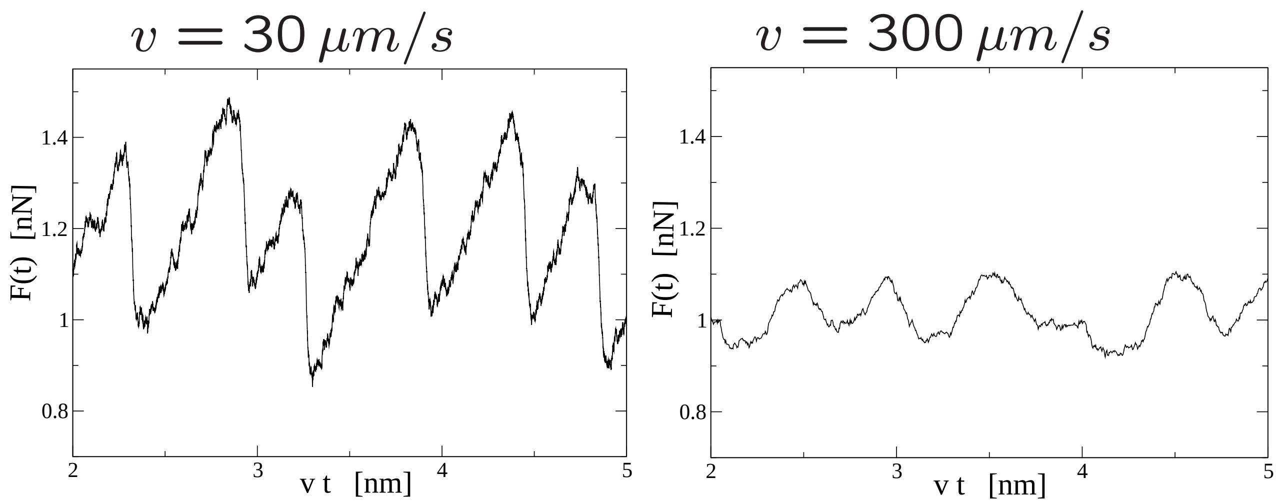

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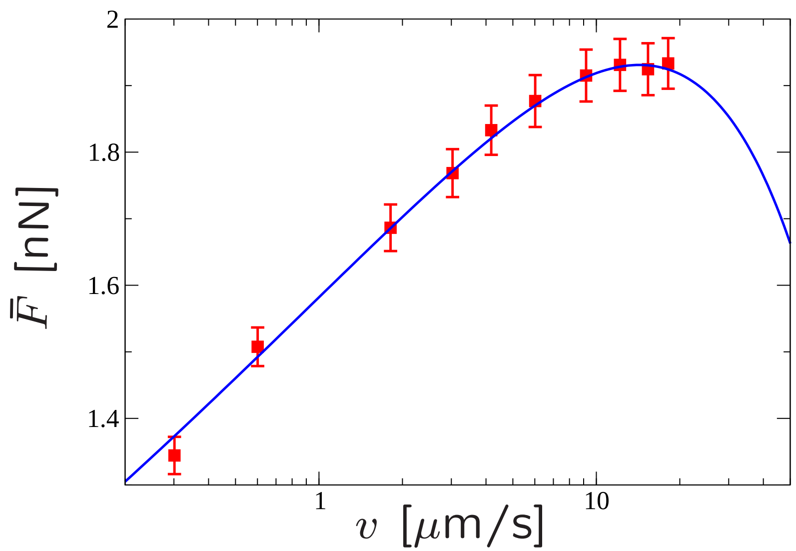
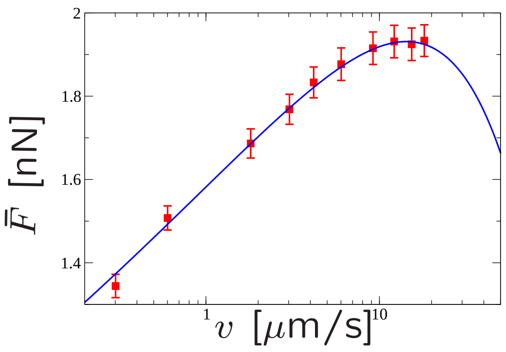
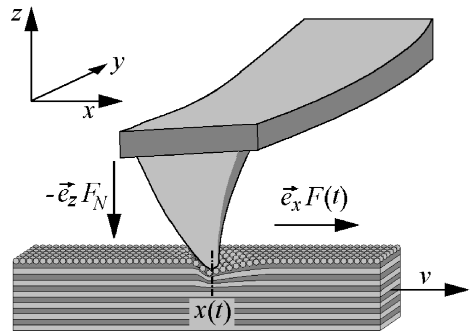


[P.R. & Evstigneev, New J. Phys. **7**, 25 (2005)]

Feature Articles





- D. Bradley, **IOP press release**, Feb. 2005

Scientists close in on "superbrakes" for cars

- L. Hutson, **Material World (London)**, March issue 2005

Breaking news

- P. Grumberg, **Science & Vie (Paris)**, March issue 2005

Un effet inattendu du frottment pourrait améliorer le freinage