

Seventh Workshop on Random Dynamical Systems

12 - 13 December 2014

Department of Mathematics University of Bielefeld Room V2–210/216

This workshop is part of the conference programme of the DFG-funded CRC 701 Spectral Structures and Topological Methods in Mathematics at the Faculty of Mathematics at the University of Bielefeld



Organizer: Barbara Gentz

http://www.math.uni-bielefeld.de/~gentz/pages/WS14/RDS14/RDS14.htm

Programme

Friday, 12 December 2014

- 8:30 9:00 Registration and coffee in V3–201
- 9:00 9:10 Friedrich Götze (Speaker of the CRC 701) Opening of the workshop
- 9:10 9:55 **Reinhard Höpfner** (Johannes-Gutenberg-Universität Mainz) Harris recurrence for strongly degenerate stochastic systems, with application to stochastic Hodgkin–Huxley models
- 10:05–10:50 **Christian Kuehn** (TU Wien) Neural fields, finite-dimensional approximation, and large deviations
- 10:50-11:20 Coffee break in V3-201
- 11:20-12:05 **Christophe Poquet** (Tor Vergata University, Roma) Subgeometric convergence to non-equilibrium stationary states for coupled rotors
- 12:15–13:00 **Nils Berglund** (Université d'Orléans) Extreme-value theory and the stochastic exit problem
- 13:00-14:15 Lunch break
- 14:15–15:00 **Viorel Barbu** (Romanian Academy & Al. I. Cuza University, Iasi) The "sliding mode" dynamics for stochastic differential equations driven by linear multiplicative noise
- 15:10–15:55 **Ioana Ciotir** (LMI, INSA Rouen) Self-repelling diffusions via an infinite dimensional approach
- 15:55–16:25 Coffee break in V3–201
- 16:25–17:10 **Jonas Tölle** (Universität Bielefeld) Stability and rescaling of singular nonlinear SPDEs with nonlocal drift
- 17:20–18:05 **L'ubomír Baňas** (Universität Bielefeld) Numerical approximation of the stochastic wave map equation
- 19:00 Joint dinner in the city centre at Zwanzig Dreizehn (Klosterplatz 13, 33602 Bielefeld, ☎ +49 521 453 642 44)

Please note: For the dinner, prior registration is required.

All talks will take place in V2–210/216.

Saturday, 13 December 2014

Coffee in V3–201
Sabine Jansen (Ruhr-Universität Bochum) Berman–Konsowa principle for reversible Markov jump processes
Oriane Blondel (Université Claude Bernard Lyon 1) L_2 -perturbed Markov processes and random walks in dynamic random environments
Coffee break in V3–201
Noemi Kurt (TU Berlin) A population model with geometric seed-bank component
Lunch break
Ilya Pavlyukevich (Friedrich-Schiller-Universität Jena) Metastable behaviour of a discrete dynamical system driven by a stable Lévy process

14:10–14:55 **Dirk Blömker** (Universität Augsburg) Stochastic front motion and slow manifolds

Abstracts

L'ubomír Baňas (Universität Bielefeld)

Numerical approximation of the stochastic wave map equation

The wave map into sphere equation serves as a prototype for wave map models that arise in various physical applications, such as, general relativity or particle physics. We propose a stochastic variant of the wave map equation; the equation is perturbed by a nonlinear multiplicative Stratonovich noise which preserves the properties of the deterministic model. We introduce a structure preserving numerical approximation of the stochastic wave map equation and present some numerical experiments to illustrate asymptotic behavior of the model.

The talk is based on a joint work with Z. Brzezniak, M. Neklyudov, M. Onderjat and A. Prohl.

Viorel Barbu (Romanian Academy & Al. I. Cuza University, Iasi)

The "sliding mode" dynamics for stochastic differential equations driven by linear multiplicative noise

For stochastic control systems of the form

$$\mathrm{d}X + AX\,\mathrm{d}t = \sigma(X)\,\mathrm{d}W + u\,\mathrm{d}t$$

one constructs feedback discontinuous controllers u = F(X) which induce a "sliding mode" dynamics in corresponding closed loop system.

Nils Berglund (MAPMO–CNRS, Université d'Orléans)

Extreme-value theory and the stochastic exit problem

Recently, F. Cérou, A. Guyader, T. Lelièvre, and F. Malrieu proved that the Gumbel distribution, which is one of the max-stable distributions known from extreme-value theory, governs the length of reactive paths in one-dimensional bistable stochastic differential equations. While the deep reason for this connection is not yet fully understood, partial progress in this direction has been achieved by Y. Bakhtin. I will present some of these insights, as well as an extension to a two-dimensional situation, with an application to noise-induced phase slips between synchronised oscillators.

Partly based on joint work with Barbara Gentz (Bielefeld).

Dirk Blömker (Universität Augsburg)

Stochastic front motion and slow manifolds

We consider several examples of spatially one-dimensional stochastic partial differential equations like the Cahn–Hilliard equation perturbed by additive noise, and study the dynamics of interfaces for the stochastic model.

The dynamic of the stochastic infinite dimensional system is given by the motion along a finite dimensional deterministic slow manifold M that is parametrized by the interface positions. Main results include the stochastic stability for M, and an explicit derivation of an effective equation for the interface positions.

Joint work with Dimitra Antonopoulou and Georgia Karali.

Oriane Blondel (Université Claude Bernard Lyon 1)

 ${\it L}_2\mbox{-}perturbed$ Markov processes and random walks in dynamic random environments

We prove general expansion results for L_2 perturbations of Markov processes generators with positive spectral gap. This gives us a toolbox for the study of certain random walks in dynamic random environments, when the random walks are "only slightly" asymmetric. We study in particular the invariant measure of the process seen from the walker and prove an invariance principle for its trajectory.

This is a joint work with Luca Avena (Leiden University) and Alessandra Faggionato (La Sapienza, Rome).

Ioana Ciotir (LMI, INSA Rouen)

Self-repelling diffusions via an infinite dimensional approach

In the present work we study self-interacting diffusions following an infinite dimensional approach. First we prove existence and uniqueness of a solution with Markov property. Then we study the corresponding transition semigroup and, more precisely, we prove that it has Feller property and we give an explicit form of an invariant probability of the system.

Joint paper with Michel Benaïm, Carl-Erik Gauthier.

Reinhard Höpfner (Johannes-Gutenberg-Universität Mainz)

Harris recurrence for strongly degenerate stochastic systems, with application to stochastic Hodgkin–Huxley models

We consider strongly degenerate stochastic differential equations having analytic coefficients, a diffusion coefficient which does not depend on time, and a drift depending on both time and spatial position which is periodic in the time argument. Our aim is to give simple criteria for positive Harris recurrence. These are formulated in terms of control systems and the support theorem, in terms of one inner point of the state space which is of full weak Hoermander dimension, and in terms of some Lyapunov function.

As an application, we can consider a stochastic Hodgkin–Huxley model for a spiking neuron where dendritic input – carrying some deterministic periodic signal coded in its drift coefficient – is the only source of noise. This amounts to a 5d SDE driven by 1d Brownian motion for which we can prove positive Harris recurrence. This approach provides us with laws of large numbers which allow to describe the spiking activity of the neuron in the long run.

Joint work with Eva Löcherbach (Université Cergy-Pontoise) and Michele Thieullen (Université Paris VI).

Sabine Jansen (Ruhr-Universität Bochum)

Berman–Konsowa principle for reversible Markov jump processes

We prove a version of the Berman–Konsowa principle for reversible Markov jump processes on Polish spaces. The Berman–Konsowa principle provides a variational formula for the capacity of a pair of disjoint measurable sets. The Berman–Konsowa principle complements the Dirichlet principle and the Thomson principle, and turns out to be especially useful for obtaining sharp estimates on crossover times in metastable interacting particle systems.

Joint work with Frank den Hollander.

Christian Kuehn (TU Wien)

Neural fields, finite-dimensional approximation, and large deviations

In this talk, I shall outline some finite-dimensional approximation results for stochastic neural fields, which are infinite-dimensional partial-integro differential equations perturbed by a Q-trace-class Wiener process. Furthermore, I will connect the results to large deviation theory and numerical schemes for stochastic bifurcation analysis. In particular, I am going to explain a new paradigm that intertwines stochastic analysis and efficient numerical parameter studies of large classes of stochastic differential equation models. The first part of this work is joint work with Martin Riedler. The talk is based upon the three papers:

- [1] C. Kuehn, C., & M.G. Riedler: Large deviations for nonlocal stochastic neural fields, The Journal of Mathematical Neuroscience 4(1) (2014), 1
- [2] C. Kuehn: Deterministic continuation of stochastic metastable equilibria via Lyapunov equations and ellipsoids, SIAM Journal on Scientific Computing 34(3) (2012), A1635–A1658
- [3] C. Kuehn: Numerical continuation and SPDE stability for the 2D cubic-quintic Allen-Cahn equation, Preprint, arXiv:1408.4000.

Noemi Kurt (TU Berlin)

A population model with geometric seed-bank component

We introduce a new model for a population where individuals may take 'dormant forms', and identify a new natural coalescent structure, the seed-bank coalescent, which describes the gene genealogy of such populations. The qualitatively new feature of the seed-bank coalescent is that ancestral lineages are independently blocked at a certain rate from taking part in coalescence events, thus strongly altering the predictions of classical coalescent models. We discuss the long-time behaviour of the population model and the corresponding coalescent, and show that even thought fixation of one type happens almost surely, the time to fixation is much longer than in classical population models. In the retrospective picture, we show that, the seed-bank coalescent 'does not come down from infinity', and the time to the most recent common ancestor is highly elevated. This provides a genealogical explanation for the empirical observation that seed-banks drastically increase genetic variability in a population and indicates how they may serve as a buffer against other evolutionary forces such as genetic drift and selection.

Joint work with J. Blath, A. Gonzalez Casanova, M. Wilke Berenguer (all TU Berlin).

Ilya Pavlyukevich (Friedrich-Schiller-Universität Jena)

Metastable behaviour of a discrete dynamical system driven by a stable Lévy process

We consider a finite state time discrete Markov chain that is obtained from the Euler approximation of the stochastic differential equation $X_t^{\varepsilon}(x) = x - \int_0^t U'(X_s^{\varepsilon}) ds + \varepsilon L_t$, where U is a multi-well potential with $n \ge 2$ local minima and $L = (L_t)_{t\ge 0}$ is a symmetric α -stable Lévy process. We investigate the spectrum of the generator of this chain in the limit $\varepsilon \to 0$ and localize the top n eigenvalues $\lambda_1^{\varepsilon}, \ldots, \lambda_n^{\varepsilon}$.

This is a joint work with T. Burghoff (Institute of Medical Biometry and Statistics, University of Lübeck)

Christophe Poquet (Tor Vergata University, Roma)

Subgeometric convergence to non-equilibrium stationary states for coupled rotors

We will consider a chain composed of three coupled rotors, attached to thermal baths (with possibly different temperatures) at each extremity. An important feature of this system is that when the middle rotor oscillates rapidly, the energy of this rotor decreases very slowly, due to averaging phenomena. We will construct an effective dynamics for the middle rotor, using averaging techniques, which will allow us to prove the ergodicity of the process at subgeometric rate.

This is a work in collaboration with N. Cuneo and J.-P. Eckmann.

Jonas Tölle (Universität Bielefeld)

Stability and rescaling of singular nonlinear SPDEs with nonlocal drift

We are studying SPDEs with nonlinear, singular drift term which is given by a convolution type nonlocal *p*-Laplacian integral operator. We obtain existence and uniqueness of solutions in the sense of stochastic variational inequalities. For additive Gaussian forcing, under rescaling of the kernels, we prove the convergence of solutions and the RDS to the local SPDE. We also obtain convergence of invariant distributions of the associated stochastic flows.

Joint work with Benjamin Gess (Chicago).

Registered participants

Daniel Altemeier	Universität Bielefeld
Johannes Blank	Universität Münster
L'ubomír Baňas	Universität Bielefeld
Viorel Barbu	Romanian Academy & Al. I. Cuza University, Iasi
Nils Berglund	MAPMO–CNRS, Université d'Orléans
Wolf-Jürgen Beyn	Universität Bielefeld
Philippe Blanchard	Universität Bielefeld
Dirk Blömker	Universität Augsburg
Oriane Blondel	Université Claude Bernard Lyon 1
Ioana Ciotir	LMI, INSA Rouen
Barbara Gentz	Universität Bielefeld
Friedrich Götze	Universität Bielefeld
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Lipika Kabiraj	TU Berlin
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Diana Putan	Universität Bielefeld
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Michael Röckner	Universität Bielefeld
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Aditya Saurabh	TU Berlin
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(as of 7 December 2014)