

Workshop “Gemometric analysis on manifolds, fractals and metric spaces”

March 1–March 3, 2022
Yamagata University, Japan
(All talks will be deliverd via Zoom)

March 1, 2022

9:00-9:50 (16:00-16:50 (China), 17:00-17:50(Japan))

Speaker : Qingsong Gu (Nanjing University)

Title : Superlinear elliptic inequalities on weighted graphs

Abstract : Let (V, μ) be an infinite, connected, locally finite weighted graph. We study the problem of existence or non-existence of positive solutions to a semi-linear elliptic inequality

$$\Delta u + u^\sigma \leq 0 \quad \text{in } V,$$

where Δ is the standard graph Laplacian on V and $\sigma > 0$. For $\sigma \in (0, 1]$, the inequality admits no nontrivial positive solution. For $\sigma > 1$, assuming condition (p_0) on (V, μ) , we obtain a sharp condition for nonexistence of positive solutions in terms of the volume growth of the graph, that is

$$\mu(o, n) \lesssim n^{\frac{2\sigma}{\sigma-1}} (\ln n)^{\frac{1}{\sigma-1}}$$

for some $o \in V$ and all large enough n . For any $\varepsilon > 0$, we can construct an example on a homogeneous tree $\mathbb{T}_N$ with $\mu(o, n) \approx n^{\frac{2\sigma}{\sigma-1}} (\ln n)^{\frac{1}{\sigma-1} + \varepsilon}$, and a solution to the inequality on $(\mathbb{T}_N, \mu)$ to illustrate the sharpness of $\frac{2\sigma}{\sigma-1}$ and $\frac{1}{\sigma-1}$. This talk is based on a joint work with Xueping Huang and Yuhua Sun.

10:00-10:50 (17:00-17:50(China), 18:00-18:50 (Japan))

Speaker : Philipp Suerig (University of Bielefeld, Germany)

Title : Heat kernel estimates on manifolds with ends

Abstract : We investigate heat kernel estimates of the form $p_t(x, x) \geq c_x t^{-\alpha}$, for large enough t , where α and c_x are positive reals and c_x may depend on x , on manifolds having at least one end with a polynomial volume growth.

11:00-11:50 (18:00-18:50(China), 19:00-19:50 (Japan))

Speaker : Jean-Philippe Anker (University of Orleans, France)

Title : Asymptotic behavior of solutions to the heat equation on noncompact symmetric spaces

Abstract : Consider the heat equation on $\mathbb{R}^d$

$$\begin{cases} \partial_t u(t, x) = \Delta_x u(t, x) \\ u(0, x) = f(x) \end{cases}$$

with initial data $f \in L^1(\mathbb{R}^d)$. Denote by $M = \int_{\mathbb{R}^d} f(x) dx$ the mass and by $p_t(x) = (4\pi t)^{-\frac{d}{2}} e^{-\frac{|x|^2}{4t}}$ the heat kernel. Then the following asymptotics are known to hold in $L^p(\mathbb{R}^d)$, for all $1 \leq p \leq \infty$:

$$\lim_{t \rightarrow +\infty} t^{\frac{n}{2p'}} \|u(t, \cdot) - M p_t\|_{L^p(\mathbb{R}^d)} = 0.$$

Analogous heat asymptotics may hold or not on Riemannian manifolds. In our talk, we shall discuss noncompact symmetric spaces, generalizing earlier results of J.L. Vázquez on hyperbolic spaces. This is joint work [arXiv:2112.01323] with E. Papageorgiou and H.-W. Zhang.

11:50 - (18:50 - (China), 19:50 - (Japan))

Free discussion

March 2, 2022

9:00-9:50 (16:00-16:50 (China), 17:00-17:50(Japan))

Speaker : Xiaoxiang Chai (Korea Institute for Advanced Study (KIAS))

Title : Mixed boundary value problems in Gromov dihedral rigidity

Abstract : A mixed boundary value problem is known to have lower regularity than classical case of pure Dirichlet or Neumann boundary value problems. We assume that a domain consists of pieces which meet each other forming right angles, we are able to solve the mixed boundary value problem in $C^{2,\alpha}$ sense via blow up method of L. Simon and continuity method. We apply the regularity theory to reprove a special case of Gromov dihedral conjecture. This is joint work with Inkang Kim (KIAS).

10:00-10:50 (17:00-17:50(China), 18:00-18:50 (Japan))

Speaker : Atsushi Inoue(Hokkaido University, Japan)

Title : Essential self-adjointness of the Laplacian of a rotationally symmetric Riemannian manifold

Abstract : We study the essential self-adjointness of the Laplacian on a rotationally symmetric manifold, which does not need to be geodesically complete. We establish a characterization of the essentially self-adjointness in terms of a capacity of the Cauchy boundary. Moreover, we have a sufficient condition of the essentially self-adjointness by the Minkowski codimension of the Cauchy boundary.

11:00-11:50 (18:00-18:50(China), 19:00-19:50 (Japan))

Speaker : Simon Nowak (University of Bielefeld, Germany)

Title : Sobolev regularity for nonlocal equations with VMO coefficients

Abstract : We present some Sobolev regularity results of Calderón-Zygmund-type for non-linear nonlocal equations with possibly discontinuous coefficients of VMO-type. While for corresponding local elliptic equations with VMO coefficients it is only possible to obtain higher integrability, in our nonlocal setting we are able to also prove a substantial amount of higher differentiability. Therefore, our results are in some sense of purely nonlocal type, following a recent trend of such results in the literature.

11:50 - (18:50 - (China), 19:50 - (Japan))

Free discussion

March 3, 2022

9:00-9:50 (16:00-16:50 (China), 17:00-17:50(Japan))

Speaker : Masayuki Aino (RIKEN, Japan)

Title : Convergence of Laplacian Eigenmaps and its rate for submanifolds not necessarily smooth

Abstract : Laplacian Eigenmaps is a method of manifold learning, an approach to dimensionality reduction, using eigenfunctions of the Laplacian. When data points are randomly obtained on a submanifold of Euclidean space satisfying appropriate assumptions, the continuum limit of Laplacian Eigenmaps gives the eigenfunctions of the Laplacian. Such studies have been carried out under the assumption that a quantity called reach is bounded from below for submanifolds. However, a submanifold with non-differentiable points cannot be approximated by a sequence of submanifolds satisfying the uniform lower bound of the reach. In this talk, we will discuss the convergence of Laplacian Eigenmaps and its rate when the reach assumption is replaced by a weaker assumption that allows non-differentiable points to appear in the limit.

10:00-10:50 (17:00-17:50(China), 18:00-18:50 (Japan))

Speaker : Huabin Ge (Renmin University of China, China)

Title : On Thurston's "geometric ideal triangulation" conjecture

Abstract : Using combinatorial Ricci flow methods, we shall prove the following theorem: Let M be a compact 3-manifold with boundary consisting of surfaces of genus at least 2. If M admits an ideal triangulation with valence at least 10 at all edges, then there exists a unique hyperbolic metric on M with totally geodesic boundary under which the ideal triangulation is geometric. For a special class of 3-manifolds, the theorem affirms a folklore conjecture which exists for almost 40 years under not so strong assumptions. This is based on joint work with Ke Feng and Bobo Hua.

11:00-11:50 (18:00-18:50(China), 19:00-19:50 (Japan))

Speaker : Leandro Pessoa (Universidade Federal do Piau -UFPI, Brazil, and University of Bielefeld, Germany)

Title : The intersection problem for 1-surfaces of the hyperbolic 3-space

Abstract : We will start the talk with a brief discussion on the roots of the intersection problem in Differential Geometry. After presenting some classic and recent results for minimal surfaces in the Euclidean 3-space, we will talk about some classic and new developments about this problem for 1-surfaces of the hyperbolic 3-space. We will focus on the classes of 1-surfaces with bounded curvature, recurrent 1-surfaces, and stochastically complete H -surfaces with $H < 1$.

11:50 - (18:50 - (China), 19:50 - (Japan))

Free discussion