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1. Let M be any topological space. Let K be a compact subset of M and F be a closed subset of M. Prove that if $F \subset K$ then F is compact.

2. A topological space M is called *Hausdorff* if, for any two disjoint points $x, y \in M$, there exist two disjoint open sets $U, V \subset M$ such that $x \in U$ and $y \in V$. Prove the following properties of a compact subset K of a Hausdorff topological space M.

   (a) For any $x \in K^c$ there exists an open set W_x containing x and disjoint from K.

   (b) K is a closed subset of M.

3. Let X, Y be two topological spaces and $f : X \to Y$ be a continuous mapping. Prove that if K is a compact subset of X then $f(K)$ is a compact subset of Y.

4. Prove that, on any C-manifold M, there exists a countable sequence $\{\Omega_k\}$ of relatively compact open sets such that $\Omega_k \Subset \Omega_{k+1}$ (that is, Ω_k is relatively compact and $\overline{\Omega}_k \subset \Omega_{k+1}$) and the union of all Ω_k is M. Prove also that if M is connected then the sets Ω_k can also be taken connected.

   Remark. An increasing sequence $\{\Omega_k\}$ of open subsets of M whose union is M, is called an *exhaustion sequence*. If in addition $\Omega_k \Subset \Omega_{k+1}$ then the sequence $\{\Omega_k\}$ is called a *compact exhaustion sequence*.