

Complex Analysis: Exercise 1

1. What is \sqrt{i} ?
2. Show that the function $f(z) = 1/z$ is analytic in the region $\mathbb{C} \setminus \{0\}$.
3. For which $z \in \mathbb{C}$ do the following infinite sums converge?

(a)

$$\sum_{n=0}^{\infty} \left(\frac{z}{z+1} \right)^n$$

(b)

$$\sum_{n=0}^{\infty} n^z$$

4. Let $G \subset \mathbb{C}$ be an open subset of the complex plane. Given $u, v \in G$, a *path* in G from u to v is a continuous mapping of the unit interval $\gamma : [0, 1] \rightarrow G$ such that $\gamma(0) = u$ and $\gamma(1) = v$. Let $z_0 \in G$ be some particular point in G . Let G_0 be the set of points w in G such that there exists a path in G from z_0 to w . Let G_1 be the set of points of G which are not in G_0 . (That is, $G_1 = G \setminus G_0$.) Show that both G_0 and G_1 are open sets in \mathbb{C} .