Complex Analysis: Exercise 5

1. Let
$$P(z) = \sum_{k=0}^{n} a_k z^k$$
be a polynomial of degree $n \geq 1$ (so that $a_n \neq 0$). We have seen that there exists an $R > 0$ such that
$$|P(z)| \geq |z|^n \frac{|a_n|}{2}$$
for $|z| \geq R$. Going in the other direction, does there exist an $r > 0$ and a constant $M > 0$ such that
$$\left| P\left(\frac{1}{z}\right) \right| \leq M$$
for $|z| \geq r$?

2. We have seen that two analytic functions f and g defined in a common region $G \subset \mathbb{C}$ are identical if the set of points where f and g correspond (that is, the set $\{z \in G : f(z) = g(z)\}$) has an accumulation point *within* G.

   But, for example, thinking about the set of real numbers
$$\{1/n : n \geq 2\} \subset (0, 1)\},$$
   we see that there can be an accumulation point *outside* G. Can you find two *different* analytic functions in a region $G \subset \mathbb{C}$ such that the set $\{z \in G : f(z) = g(z)\}$ has an accumulation point in $\mathbb{C}$?

3. What is
$$\int_{|z|=1} \log z dz\,?$$