

## Complex Analysis: Exercise 9

1. Calculate the following integrals using the calculus of residues.

(a)

$$\int_0^{\infty} \frac{x^2}{x^4 + 6x^2 + 13} dx$$

(b)

$$\int_0^{\infty} \frac{x \sin x}{x^2 + a^2} dx, \quad a > 0$$

(c)

$$\int_0^{\infty} \frac{\sqrt{x}}{1 + x^2} dx$$

2. Let  $g, h : \mathbb{C} \rightarrow \mathbb{C}$  be entire functions, so that  $R(z) = g(z)/h(z)$  is a rational function. Assume  $R$  has a simple zero at  $\infty$  and no poles on the real number line. Show that

$$\lim_{r \rightarrow \infty} \int_{-r}^{+r} R(x) dx$$

exists, but it does not equal

$$2\pi i \sum_{\operatorname{Im} a > 0} \operatorname{Res}_a R(z).$$

What is it equal to?