Relative homological algebra in triangulated categories

A tribute to Michael Butler

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Universität Bielefeld

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Short Curriculum Vitae
M. C. R. Butler
May 18, 2005

Full Name	Michael Charles Richard BUTLER.
Date, place of birth	6 January 1929, Melbourne, Australia.
Nationality	British.
Work address	Department of Mathematical Sciences, University of Liverpool, Liverpool, L69 7ZL, UK.
Home address	37 Sydenham Avenue, Liverpool, L17 3AU, UK.
Home telephone, email	44-(0)151-734-0034, mcrb@liv.ac.uk.
Academic Degrees	B.Sc.(1949),M.A.(1951),Ph.D.(1955), all from the University of Melbourne, Australia.
Status	Retired. Honorary Senior Fellow in Pure Mathematics, University of Liverpool, since 1997.
Employment record	Lecturer, Senior Lecturer and Reader in Pure Mathematics, University of Liverpool, 1957-96; Head of Department, 1983-88.
Research interests	Homological algebra, abelian groups, representation theory of finite-dimensional algebras and orders.

MICHAEL BUTLER

Dr Michael Charles Richard Butler, who was elected a member of the London Mathematical Society on 15 December 1955, died on 18 December 2012, aged 83.

Peter Giblin writes (with advice from Mary Rees and Claus Ringel): Michael and his wife Sheila Brenner, who died in 2002, were active members of the mathematics departments of the University of Liverpool from the time of their appointments in 1957, when they both moved from the University of London. Until the merger of the two departments (and Statistics) in the 1990s Sheila was in the Department of Applied Mathematics and Michael in the Department of Pure Mathematics; but from the early 1960s, and a joint research leave to Michael's home country of Australia, they worked together on problems in algebra.

Michael's earlier work was devoted to questions in homological algebra. His detailed study of a class of torsion-free groups of finite rank (now called Butler groups) showed the complexity of such groups. His use of representations of posets in order to study abelian groups was very influential as one of the first general reduction techniques. In several papers he described the surprising dichotomy between tame and wild behaviour of module categories. In their joint work Michael and Sheila developed 'tilting theory', now an indispensable tool in algebra and geometry providing a general framework for dealing with equivalences of triangulated categories. Their first major publication on this was in 1980: *Generalizations of the Bernstein-Gelfand-Ponomarev reflection functors*, in the proceedings of the second ICRA (International Conference on Representations of Algebras). From its beginning, Michael was one of the scientific advisors for the ICRA conference series which started in 1974 in Ottawa, Canada, and now is held every second year in different countries. Michael and Sheila's last joint publication was in 2007, five years after Sheila's death. Together, they organized a very successful symposium at the University of Durham in 1985.

Michael was a highly successful Head of the (then) Department of Pure Mathematics in Liverpool in the 1980's: perhaps surprisingly so, given his strong, forthrightly expressed, and even unfashionable, left-wing views, which were also an important part of his partnership with Sheila. But he also had exceptional organisational ability, and was naturally kind, courteous, pragmatic and level headed. Michael formally retired in 1996 but continued active in work and conference attendance until his medical condition prevented it. Michael and Sheila had no children, but Michael, from a large family, has dozens of collateral descendants, and will also be missed by his many friends around the world.

Most important publications

Citations	Publication
111	**MR0876976 (88a:16055)** Butler, M. C. R.; Ringel, Claus Michael Auslander-Reiten sequences with few middle terms and applications to string algebras. *Comm. Algebra* 15 (1987), no. 1–2, 145–179. (Reviewer: Christine Riedtmann) 16A64 (16A35)
89	**MR0607151 (83e:16031)** Brenner, Sheila; Butler, M. C. R. Generalizations of the Bernstein-Gel'fand-Ponomarev reflection functors. *Representation theory, II (Proc. Second Internat. Conf., Carleton Univ., Ottawa, Ont., 1979)*, pp. 103–169, Lecture Notes in Math., 832, Springer, Berlin-New York, 1980. (Reviewer: Idun Reiten) 16A64 (16A46)
33	**MR1930968 (2003i:16011)** Brenner, Sheila; Butler, Michael C. R.; King, Alastair D. Periodic algebras which are almost Koszul. *Algebr. Represent. Theory* 5 (2002), no. 4, 331–367. (Reviewer: Peter A. Linnell) 16E05 (16G10 16S37)
32	**MR1670674 (2000f:16013)** Butler, M. C. R.; King, A. D. Minimal resolutions of algebras. *J. Algebra* 212 (1999), no. 1, 323–362. (Reviewer: Dieter Happel) 16E40 (16G20 16G60 16G70)
29	**MR0218446 (36 #1532)** Butler, M. C. R. A class of torsion-free abelian groups of finite rank. *Proc. London Math. Soc. (3)* 15 1965 680–698. (Reviewer: W. Liebert) 20.30
15	**MR0174593 (30 #4794)** Brenner, Sheila; Butler, M. C. R. Endomorphism rings of vector spaces and torsion free abelian groups. *J. London Math. Soc.* 40 1965 183–187. (Reviewer: C. W. Curtis) 16.40
14	**MR0230767 (37 #6327)** Butler, M. C. R. Torsion-free modules and diagrams of vector spaces. *Proc. London Math. Soc. (3)* 18 1968 635–652. (Reviewer: S. B. Conlon) 16.90
13	**MR0225878 (37 #1469)** Butler, M. C. R. On locally free torsion-free rings of finite rank. *J. London Math. Soc.* 43 1968 297–300. (Reviewer: G. Michler) 20.30 (16.00)
12	**MR0188267 (32 #5706)** Butler, M. C. R.; Horrocks, G. Classes of extensions and resolutions. *Philos. Trans. Roy. Soc. London Ser. A* 254 1961/1962 155–222. (Reviewer: S. Eilenberg) 18.20

The research fields

- **Relative homological algebra**
  M.C.R. Butler, G. Horrocks: Classes of extensions and resolutions, 1961.

- **Torsion-free abelian groups**
  M.C.R. Butler, A class of torsion-free abelian groups of finite rank, 1965.

- **Representations of orders and integegral group rings**
  M.C.R. Butler: On the classification of local integral representations of finite abelian p-groups, 1974.

- **Representations of finite-dimensional algebras**
  M.C.R. Butler, C.M. Ringel: Auslander-Reiten sequences with few middle terms and applications to string algebras, 1987.

- **Tilting theory**
  S. Brenner, M.C.R. Butler: Generalizations of the Bernstein-Gel'fand-Ponomarev reflection functors, 1980.

Classifications: an early understanding of 'zahm und wild'

Classification problems (torsion-free abelian groups, poset representations, lattices over orders etc.) led to the study of tame and wild phenomena.

An exhibition 1990 in Basel.

(PRELIMINARY NOTICE)

Carleton University

Department of Mathematics

cordially invites you to an

International Conference
on
Representations of Algebras

September 3-7, 1974

(FOLLOWING THE INTERNATIONAL CONGRESS OF MATHEMATICIANS 1974 IN VANCOUVER)

TO DATE, THE FOLLOWING MATHEMATICIANS HAVE ACCEPTED A PRELIMINARY INVITATION:

M. AUSLANDER (BRANDEIS)
R. BRAUER (HARVARD)
S. BRENNER (LIVERPOOL)
M.C.R. BUTLER (LIVERPOOL)
C.W. CURTIS (OREGON)
A.W.M. DRESS (BIELEFELD)
P. GABRIEL (BONN)
H. JACOBINSKI (GÖTEBORG)
G.J. JANUSZ (ILLINOIS)
C.U. JENSEN (KOBENHAVN)
M.M. KLEINER (KIEV)

H. KUPISCH (HEIDELBERG)
G. MICHLER (GIESSEN)
L.A. NAZAROVA (KIEV)
C. PROCESI (PISA)
I. REINER (ILLINOIS)
I. REITEN (TRONDHEIM)
C.M. RINGEL (TÜBINGEN)
K.W. ROGGENKAMP (BIELEFELD)
A.V. ROITER (KIEV)
H. TACHIKAWA (TOKYO)
T. YOSHII (SHIGA)

V. DLAB
FOR THE ORGANIZING COMMITTEE OF ICRA

THOSE WISHING TO OBTAIN THE FIRST NOTICE OF THE
CONFERENCE, PLEASE WRITE TO
SECRETARY OF ICRA, DEPARTMENT OF MATHEMATICS, CARLETON UNIVERSITY, OTTAWA, CANADA

Representations of Algebras (organisers: M.C.R. Butler, S. Brenner)

Michael 1997 at a conference in Kiev.

TWENTY YEARS OF TILTING THEORY

- an Interdisciplinary Symposium -

November 18-22, 2002,
Fraueninsel, Germany

Tilting modules were born about twenty years ago in the context of finite dimensional algebras. Since then, tilting theory has spread in many different directions, and nowadays it plays an important role in various branches of modern algebra, ranging from Lie theory and algebraic geometry to homotopical algebra. The aim of this meeting is to bring together for the first time experts from different fields where tilting is relevant or even of central importance. There will be several lecture series and survey talks on the use of tilting theory in different contexts, as well as a number of additional talks contributed by the participants.

Here is a tentative list of the *invited speakers*:

M. van den Bergh (University of Limburg)
S. Brenner (University of Liverpool)
T. Brüstle (University of Bielefeld)
M. Butler (University of Liverpool)
S. Donkin (University of London)
K. Erdmann (University of Oxford)
K. Fuller (University of Iowa)
B. Keller (University of Paris VII)
S. König (University of Leicester)
H. Lenzing (University of Paderborn)
O. Mathieu (University of Lyon)
J. Miyachi (Tokyo Gakugei University)
I. Reiten (NTNU Trondheim)
J. Rickard (University of Bristol)
C. M. Ringel (University of Bielefeld)
R. Rouquier (University of Paris VII)
J. Trlifaj (Charles University Prague)

Organizers: Lidia Angeleri Hügel (Munich), Dieter Happel (Chemnitz), Henning Krause (Bielefeld).

CRUDDIS, Thomas Barry: On a class of torsion free abelian groups, 1964

SHAHZAMANIAN, Mostafa: Representation of Dynkin graphs by abelian p-groups, 1979

COELHO, Flávio Ulhoa: Preprojective partitions and Auslander-Reiten quivers for artin algebras, 1990

BURT, William Leighton: Homological theory of bocs representations, 1991

Sheila Brenner: Henning Krause, Shiping Liu
Michael Butler: Flávio Coelho, William Burt

Some of the postdocs in Liverpool in the 1980/90s

Bill Crawley-Boevey

Mike Prest

Alastair King

The communist

Shaking hands with Mao

The daily newspaper

Your comments on ICRA were interesting. It is exciting that the 'old representation theory' has become so important in applications to areas of applicable mainstream maths, a development which will keep it alive as a subject in its own right (unlike, for example, torsion free abelian group theory!!!!), and maybe lead to solutions of some of the remaining hard problems of pure reprn theory; there is an analogy here with the way 'pure complex function theory' still develops because of its vast applications. Of course it makes life harder for old men like me, but I do really like what is happening.

Again: the research fields of Michael Butler

- **Relative homological algebra**
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- **Torsion-free abelian groups**
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- **Representations of orders and integegral group rings**
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- **Tilting theory**
  S. Brenner, M.C.R. Butler: Generalizations of the Bernstein-Gel'fand-Ponomarev reflection functors, 1980.

Relative homological algebra

- The authors write: *The ideas of relative homological algebra have been formulated for categories of modules by Hochschild (1956), and for abstract categories by Heller (1958) and Buchsbaum (1959). The common feature of these papers is the selection of a class of extensions or, equivalently, a class of monomorphisms and epimorphisms. In Hochschild's paper it is the class of extensions which split over a given subring of the ring of operators.*

- Thus: Relative homological algebra is the study of an abelian category C by looking at certain subfunctors of $\mathrm{Ext}^1_C(-, -)$.

- The centre $Z(C)$ of C is introduced as the commutative ring of all endomorphisms $\mathrm{Id}_C \to \mathrm{Id}_C$ of the identity functor.

- Subfunctors of $\mathrm{Ext}^1_C(-, -)$ arise from $Z(C)$.

- Example: For a ring A, the centre $Z(\mathrm{Mod}\,A)$ is isomorphic to the centre $Z(A)$.

Relative homological algebra in triangulated categories

The basic ingredients:

- We pass from an abelian category to its derived category.
- More generally, we work in a triangulated category T.
- We work with its graded centre $Z^*(\mathsf{T})$.
- We work relative to a map $R \to Z^*(\mathsf{T})$ of graded rings.
- We consider for each graded prime ideal $\mathfrak{p}$ of R the localisation

$$\operatorname{Hom}^*_{\mathsf{T}}(-, -)_{\mathfrak{p}}.$$

The basic results:

- We obtain a local-global principle for T.
- In interesting cases, we obtain a stratification of T. Then the relative theory is independent of the choice of R.
- In other cases, the graded centre $Z^*(\mathsf{T})$ is too small.

Joint work with Dave Benson and Srikanth Iyengar
(arXiv:1305.1668)