

Representations of hereditary algebras

Exercises 4

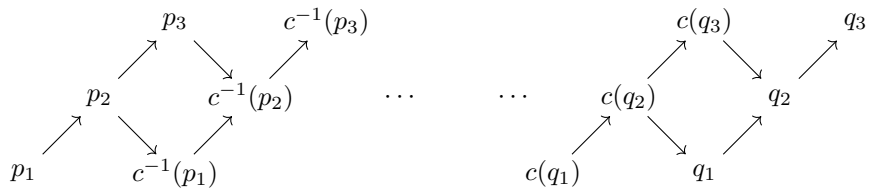
- Let K/k be a field extension of degree n , and consider the finite dimensional hereditary k -algebra

$$R = \begin{pmatrix} K & & \\ K & k & \\ K & k & k \end{pmatrix}.$$

We can write this as $R = A \oplus J$ where A is semisimple and J is a nilpotent two-sided ideal (the Jacobson radical). What is A ? What is J ? What is J^2 ?

Use this information to compute the matrix representing the Euler form, the classes $p_i = [P_i]$ of the indecomposable projectives, the classes $q_i := [I_i]$ of the indecomposable injectives, and the matrices representing the Coxeter transformation and its inverse.

We draw the classes of the indecomposable preprojectives and postinjectives as



Compute these classes when $n = 1, 2$.

Sketch what happens for $n = 3$. Can you prove that there are infinitely many classes of indecomposable preprojectives, and similarly postinjectives? Show that the Coxeter transformation has 1 as an eigenvalue. Compute a corresponding eigenvector. What is the connection to the classes of preprojectives/postinjectives?

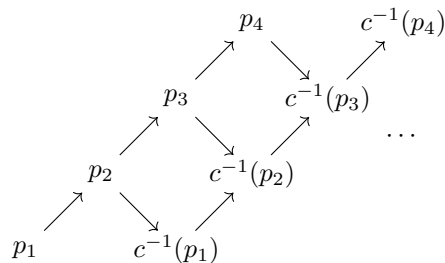
Can you find a formula for $\frac{1}{d_i}(c^{-r}(p_i) + c^{-r-1}(p_i))$ involving the classes of the other preprojectives, where $d_i := \langle p_i, p_i \rangle$.

- Give an example of a finite dimensional hereditary tensor algebra $R = T_A(M)$ such that the Grothendieck group is \mathbb{Z}^4 with Euler form given by the matrix

$$\langle -, - \rangle \leftrightarrow \begin{pmatrix} 1 & & & \\ -1 & 1 & & \\ & -2 & 2 & \\ & & -2 & 2 \end{pmatrix}.$$

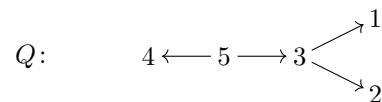
Compute the classes p_i and q_i , the matrix representing the Coxeter transformation. Thus write down all the classes of indecomposable preprojec-

tives, drawing them as



Again, can you find a formula for $\frac{1}{d_i}(c^{-r}(p_i) + c^{-r-1}(p_i))$, where $d_i := \langle p_i, p_i \rangle$.

3. Consider the quiver



Express the path algebra kQ as a matrix algebra (a subalgebra of $M_5(k)$). Compute the matrix representing the Euler form.

Draw the classes of the indecomposable preprojectives as

