Representations of hereditary algebras
Exercises 5

1. Let X, Y be indecomposable with X not regular. Show that there cannot be homomorphisms $X \to Y$ and $Y \to \tau X$ with both non-zero.

   Deduce, using the Auslander-Reiten Formula, that $Y \in X^{\perp}$ if and only if $\langle X, Y \rangle = 0$.

   Dually, $Y \in {}^{\perp}X$ if and only if $\langle Y, X \rangle = 0$.

2. Starting from a (connected) generalised Cartan lattice of affine type we can take one regular-simple S_i from each tube of period $p_i > 1$, and set $\mathcal{X} := \{\tau^j S_i : 0 \leq j < p_i - 1\}$. Then $\mathcal{X}^{\perp}$ is tame homogeneous of rank two, so has type either $\tilde{\mathbb{A}}_1$ or $\tilde{\mathbb{A}}_1'$.

   Compute for each generalised Cartan lattice of affine type with choice of regular-simples S_i on the handout the corresponding type of the rank two lattice $\mathcal{X}^{\perp}$.