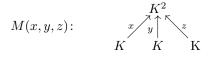
## Non-commutative Algebra, SS 2019

Lectures: W. Crawley-Boevey Exercises: A. Hubery

## Exercises 4

Throughout, K is a field.

1. Given three vectors  $x, y, z \in K^2$ , consider the representation of the three subspace quiver given by



- (a) Show that for  $a = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ,  $b = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$  and  $c = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ , the representation M(a, b, c) is indecomposable.
- (b) Suppose that x, y, z are all non-zero and span distinct lines in  $K^2$ . Show that  $M(x, y, z) \cong M(a, b, c)$ .
- (c) Suppose that x, y are non-zero and span distinct lines, and consider the two representations M(x, y, y) and M(x, y, 0). In each case give explicit decompositions into direct sums.
- 2. Consider the quiver  $1 \xrightarrow{a} 2 \xrightarrow{b} 3$ . A representation is thus given by  $U \xrightarrow{f} V \xrightarrow{g} W$ , where U, V, W are vector spaces and f, g are linear maps. By considering kernels, images and cokernels of f and g, as well as their possible intersections and unions, show that there are six finite dimensional indecomposable representations, and every representation is isomorphic to a direct sum of copies of these six indecomposable representations.
- 3. Let R be the path algebra of the quiver  $1 \xrightarrow{p} 2$ . Show that  $R \otimes_K R$  is isomorphic to the commutative square, so KQ/(ca db) for the quiver

$$\begin{array}{ccc} 1 & \stackrel{a}{\longrightarrow} & 2 \\ \downarrow_{b} & & \downarrow_{c} \\ 3 & \stackrel{d}{\longrightarrow} & 4 \end{array}$$

4. Consider the algebra  $R = KQ/KQ_+^2$  for the quiver

$$Q: 1 \xrightarrow[b]{a} 2$$

Show that dim R = 4 and compute a basis for R. Compute the representations P[i] for i = 1, 2. Show further that R is a Frobenius algebra, but is not a symmetric algebra.

To be handed in by 10th May.