Non-commutative Algebra, SS 2019

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Exercises 4

Throughout, K is a field.

1. Given three vectors $x, y, z \in K^2$, consider the representation of the three subspace quiver given by

$$M(x,y,z)\colon \qquad \begin{array}{ccccc} & & K^2 & & \\ & \overset{x}{\nearrow} & \overset{y}{\uparrow} & \overset{z}{\nwarrow} & \\ K & & K & & \mathrm{K} \end{array}$$

   (a) Show that for $a = \binom{1}{0}$, $b = \binom{0}{1}$ and $c = \binom{1}{1}$, the representation $M(a,b,c)$ is indecomposable.

   (b) Suppose that x, y, z are all non-zero and span distinct lines in K^2. Show that $M(x,y,z) \cong M(a,b,c)$.

   (c) Suppose that x, y are non-zero and span distinct lines, and consider the two representations $M(x,y,y)$ and $M(x,y,0)$. In each case give explicit decompositions into direct sums.

2. Consider the quiver $1 \xrightarrow{a} 2 \xrightarrow{b} 3$. A representation is thus given by $U \xrightarrow{f} V \xrightarrow{g} W$, where U, V, W are vector spaces and f, g are linear maps. By considering kernels, images and cokernels of f and g, as well as their possible intersections and unions, show that there are six finite dimensional indecomposable representations, and every representation is isomorphic to a direct sum of copies of these six indecomposable representations.

3. Let R be the path algebra of the quiver $1 \xrightarrow{p} 2$. Show that $R \otimes_K R$ is isomorphic to the commutative square, so $KQ/(ca - db)$ for the quiver

$$Q\colon \quad \begin{array}{ccc} 1 & \xrightarrow{a} & 2 \\ \downarrow{\scriptstyle b} & & \downarrow{\scriptstyle c} \\ 3 & \xrightarrow{d} & 4 \end{array}$$

4. Consider the algebra $R = KQ/KQ_+^2$ for the quiver

$$Q\colon \quad 1 \underset{b}{\overset{a}{\rightleftarrows}} 2$$

   Show that $\dim R = 4$ and compute a basis for R. Compute the representations $P[i]$ for $i = 1, 2$. Show further that R is a Frobenius algebra, but is not a symmetric algebra.

To be handed in by 10th May.