Non-commutative Algebra, WS 19/20

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Exercises: A. Hubery

Exercises 4

1. Knit the Auslander-Reiten quivers for the path algebras of the following quivers

$$1 \longrightarrow 2 \longrightarrow 3 \longleftarrow 4 \qquad \text{and} \qquad 1 \longrightarrow 2 \longleftarrow 3 \longrightarrow 4$$

For the vertices of the Auslander-Reiten quiver, you can just write the dimension vector of the corresponding indecomposable. (Each indecomposable will be directing, so is determined up to isomorphism by its dimension vector.)

2. Knit the Auslander-Reiten quiver for the following algebra

$$\text{quiver} \quad \begin{tikzcd} & & 3 \\ 1 \arrow[r, "a"] & 2 \arrow[ur, "b"] \arrow[r, "c"] \arrow[dr, "d"'] & 4 \\ & & 5 \end{tikzcd} \quad \text{with relation} \quad ba.$$

3. Knit the Auslander-Reiten quiver for the following algebra

$$\text{quiver} \quad \begin{tikzcd} 6 \arrow[d, "e"] & 4 \arrow[d, "c"] & 2 \arrow[d, "a"] \\ 5 \arrow[r, "d"] & 3 \arrow[r, "b"] & 1 \end{tikzcd} \quad \text{with relation} \quad bd.$$

4. Let A be the algebra given via

$$\text{quiver} \quad \begin{tikzcd} 1 \arrow[r, bend left, "a"] & 2 \arrow[l, bend left, "b"] \end{tikzcd} \quad \text{with relation} \quad aba.$$

(a) Write out a basis for A.
(b) Compute the indecomposable projective and injective modules. Deduce that A is a Nakayama algebra.
(c) Describe all indecomposable A-modules.
(d) Compute the Auslander-Reiten translate of each indecomposable module. (Note that there are two radical maps $f, g\colon P[1] \to P[2]$ and hence also two radical maps $I[1] \to I[2]$, so you need to be careful in computing $\nu(f), \nu(g)$.)
(e) Compute the Auslander-Reiten quiver of A.
(You cannot knit this quiver, but you do have enough information to construct it. For example, you should have shown $\tau(S[1]) \cong S[2]$, so we know the Auslander-Reiten sequence ending at $S[1]$, and hence all irreducible maps starting at $S[2]$ and all irreducible maps ending at $S[1]$.)

To be handed in by 18th November.